Structure of the set of all proper colourings (and a crush-course on phase transitions)

Roman Kotecký

Praha & Warwick

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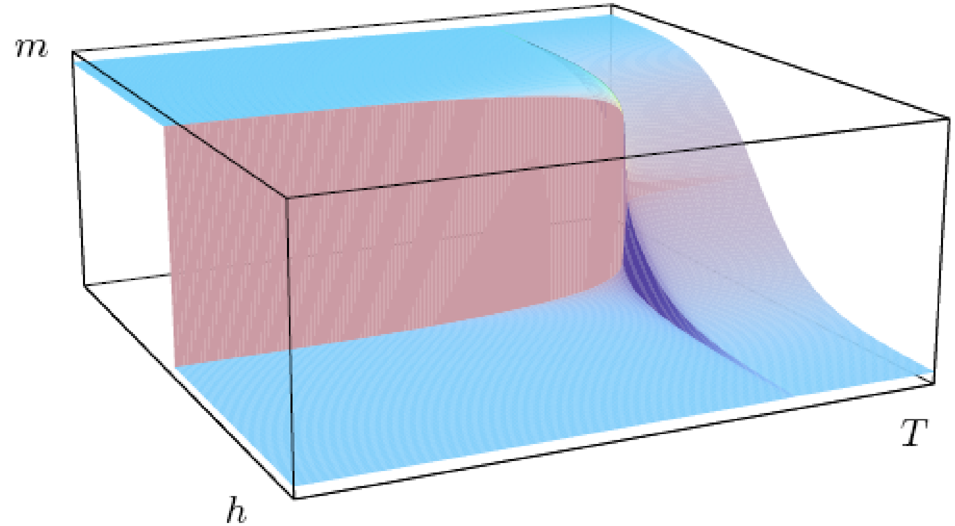

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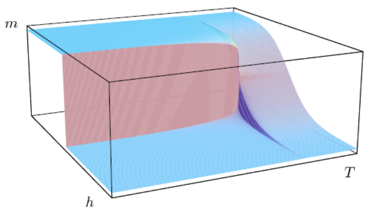

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$$\mu(\cdot)_{\overline{\sigma}} = \lim_{\Lambda_n\nearrow\mathbb{Z}^d}\mu_{\Lambda_n}(\cdot \mid \overline{\sigma})$$

An alternative formulation:

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All this makes sense for zero temperature, $\beta = \infty$,
with $\mu_\Lambda^{\beta=\infty}(\sigma_\Lambda \mid \overline{\sigma})$ being a uniform distribution over all σ_Λ
such that $H_\Lambda(\sigma_\Lambda \mid \overline{\sigma}) = 0$

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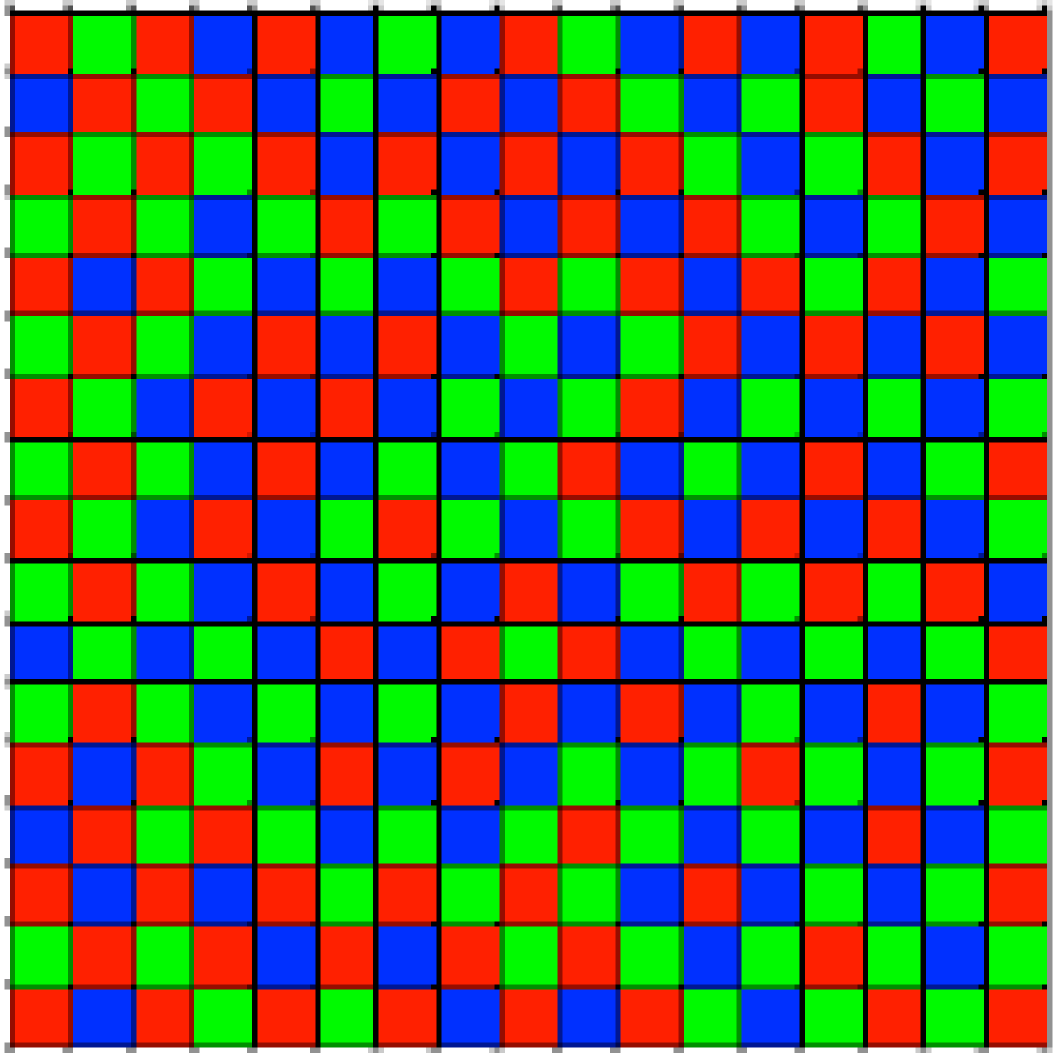
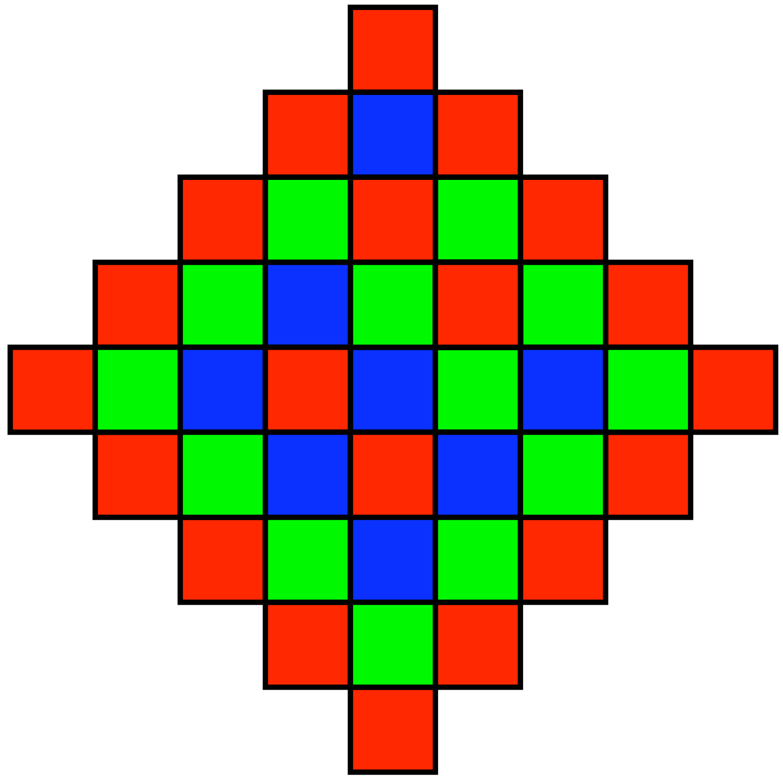
all of them uniformly





1. I have a dream

1. *I have a dream*
2. *Failure*

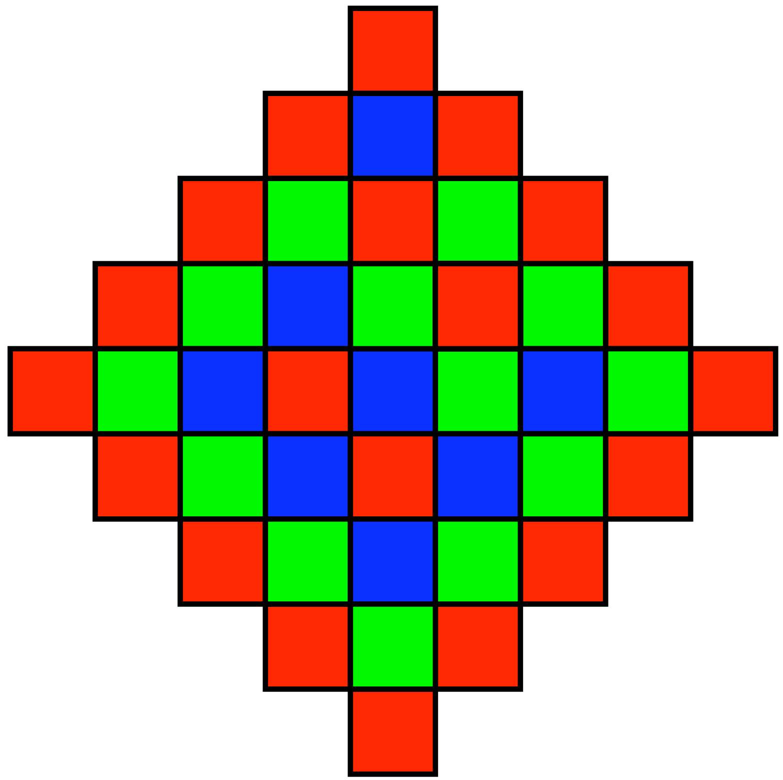
1. *I have a dream*
2. *Failure*
3. *Happy ending*

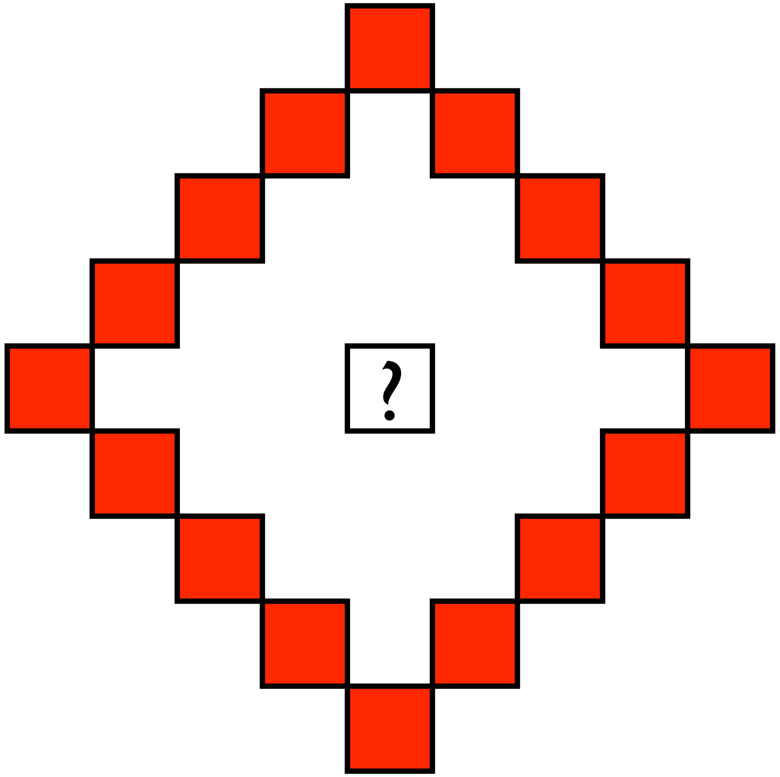
PHYSICAL REVIEW B VOLUME 31, NUMBER 5 1 MARCH 1985

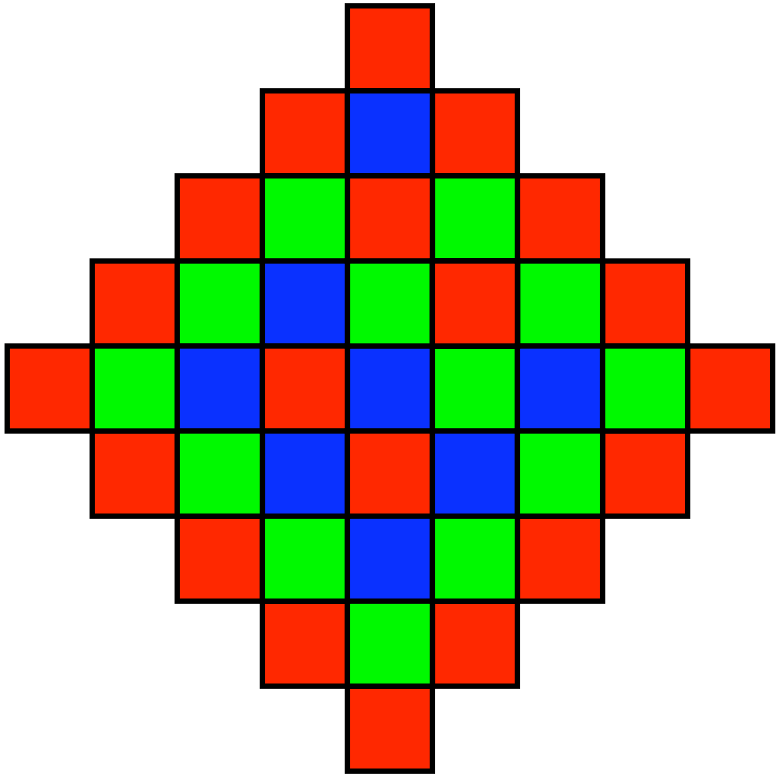
Long-range order for antiferromagnetic Potts models

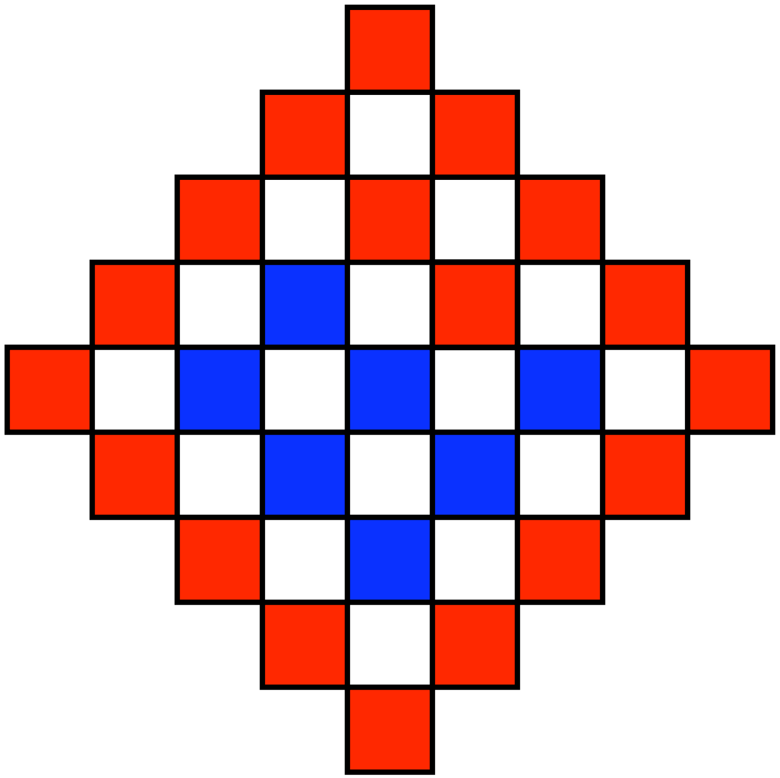
Roman Kotecký
Department of Mathematical Physics, Charles University, V Holešovičkách 2, 180 00 Praha 8, Czechoslovakia
(Received 29 March 1984; revised manuscript received 30 July 1984)

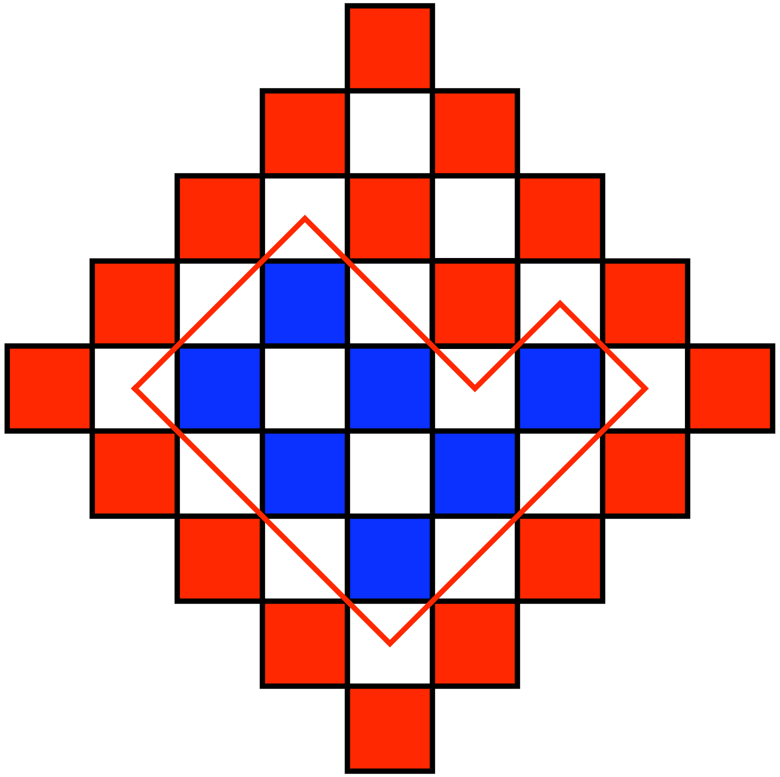
Long-range order for the three-state antiferromagnetic Potts model may appear at zero temperature as an instability with respect to boundary conditions. It is studied using an approximate correspondence, reminiscent of duality, which links this model with the ferromagnetic Ising model at a particular temperature. The basic idea is to represent entropy constraints in the former in terms of energy increase in the latter. The correspondence can be made exact by modifying the Ising model. The (non)existence of long-range order is then linked to the location of the critical temperature of the modified Ising model with respect to the particular value given by the correspondence.



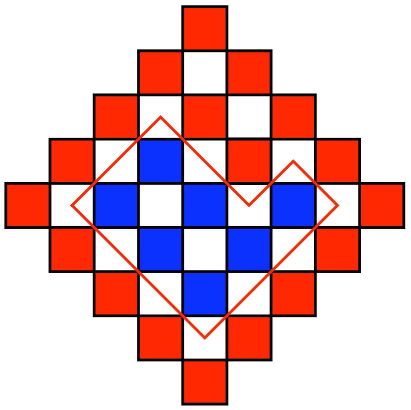




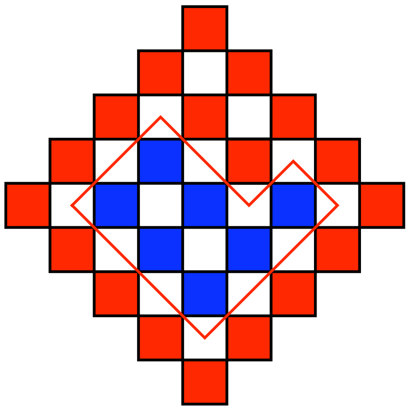




P (střed je modrý)



$$P(\text{střed je modrý}) \leq \sum_{\gamma \text{ okolo středu}} P(\gamma)$$



$$P(\text{střed je modrý}) \leq \sum_{\gamma \text{ okolo středu}} P(\gamma) \leq$$

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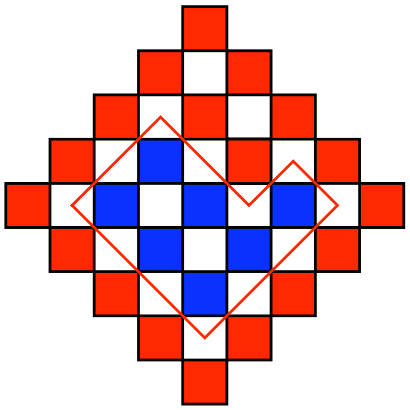

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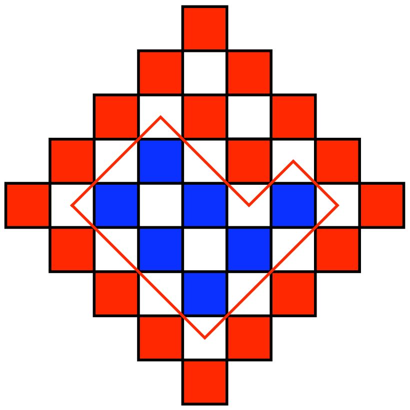

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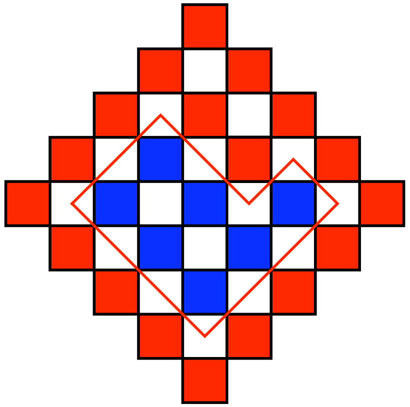

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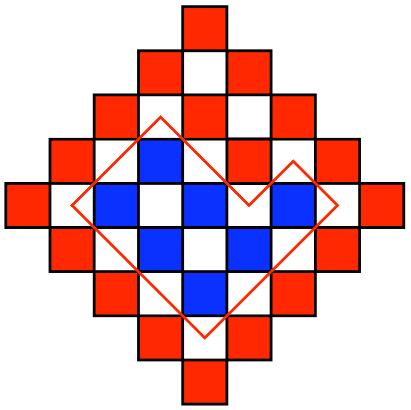

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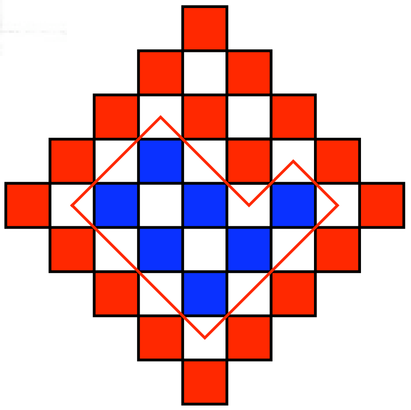

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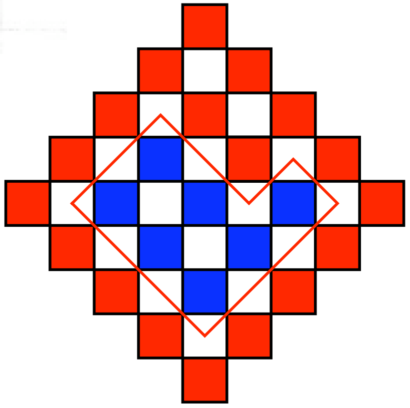
NESTAČÍ



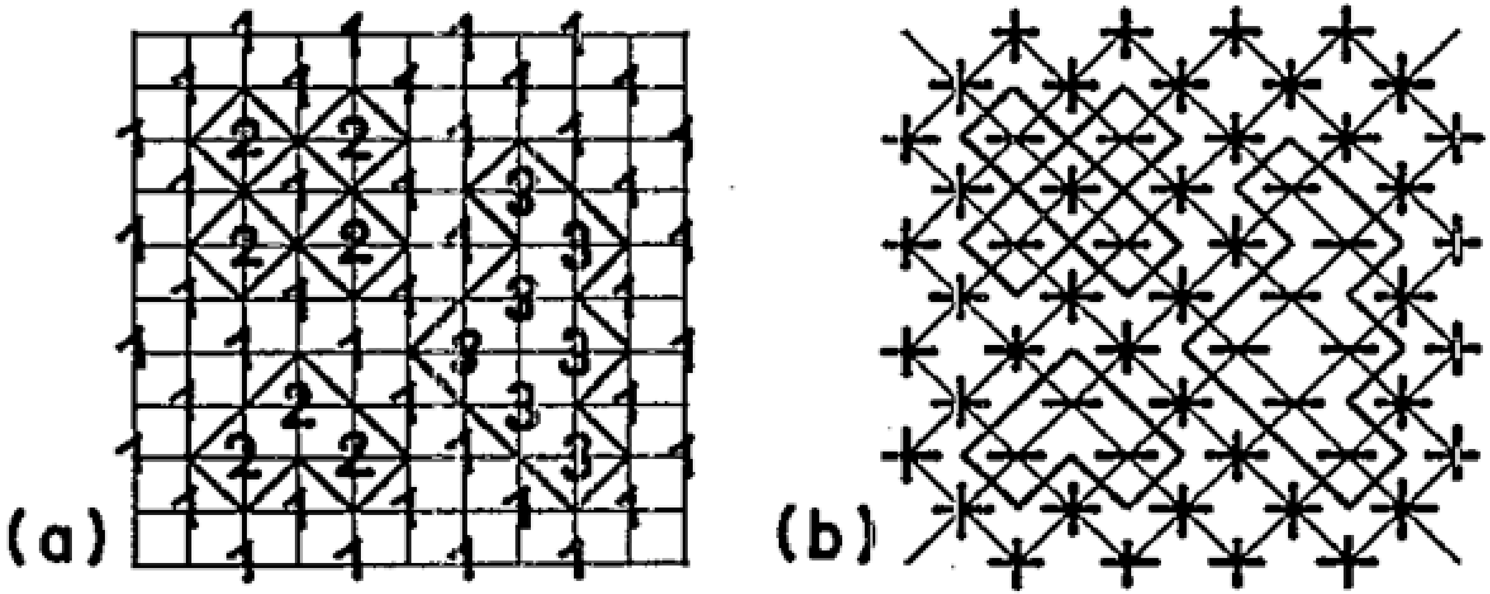


FIG. 2. Contours for (a) the AF Potts model on Λ, (b) the F Ising model on Λ_b ($+$, $-$ denotes spin up, down, respectively). For the boundary ∂ drawn, it is $|\partial|=3$, $\|\partial\|_v=34$, $\|\partial\|_e=38$, $|\partial|_c=7$, and $\|\partial\|_{v^{(4)}}=4$. Note that for the contour shown in the left upper corner, the restriction (i) implies that the spin 1 in its center is compulsory and that the only different configuration on the black sublattice consistent with it is that with 2 changed to 3 on all four sites.

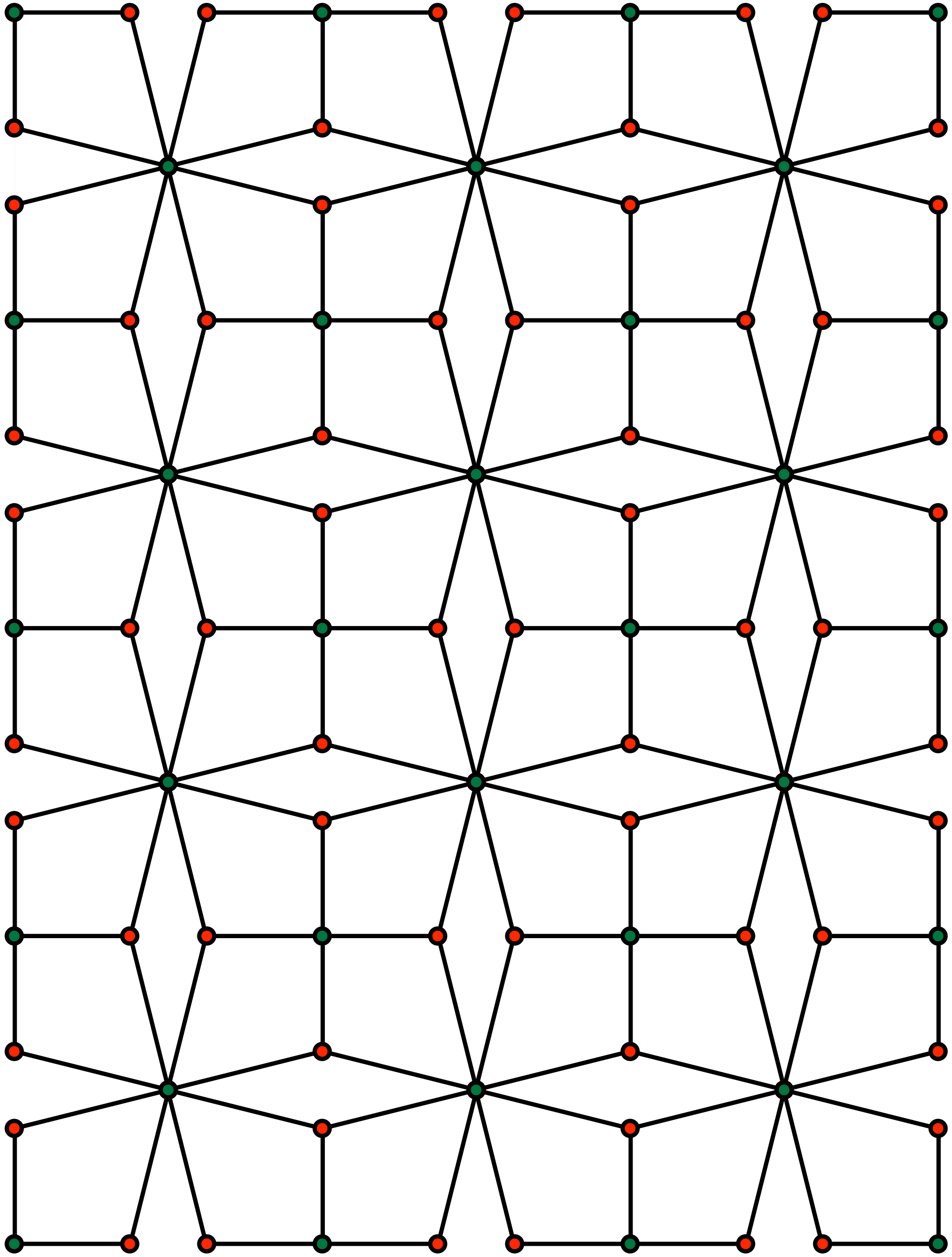
September 7th, 2000

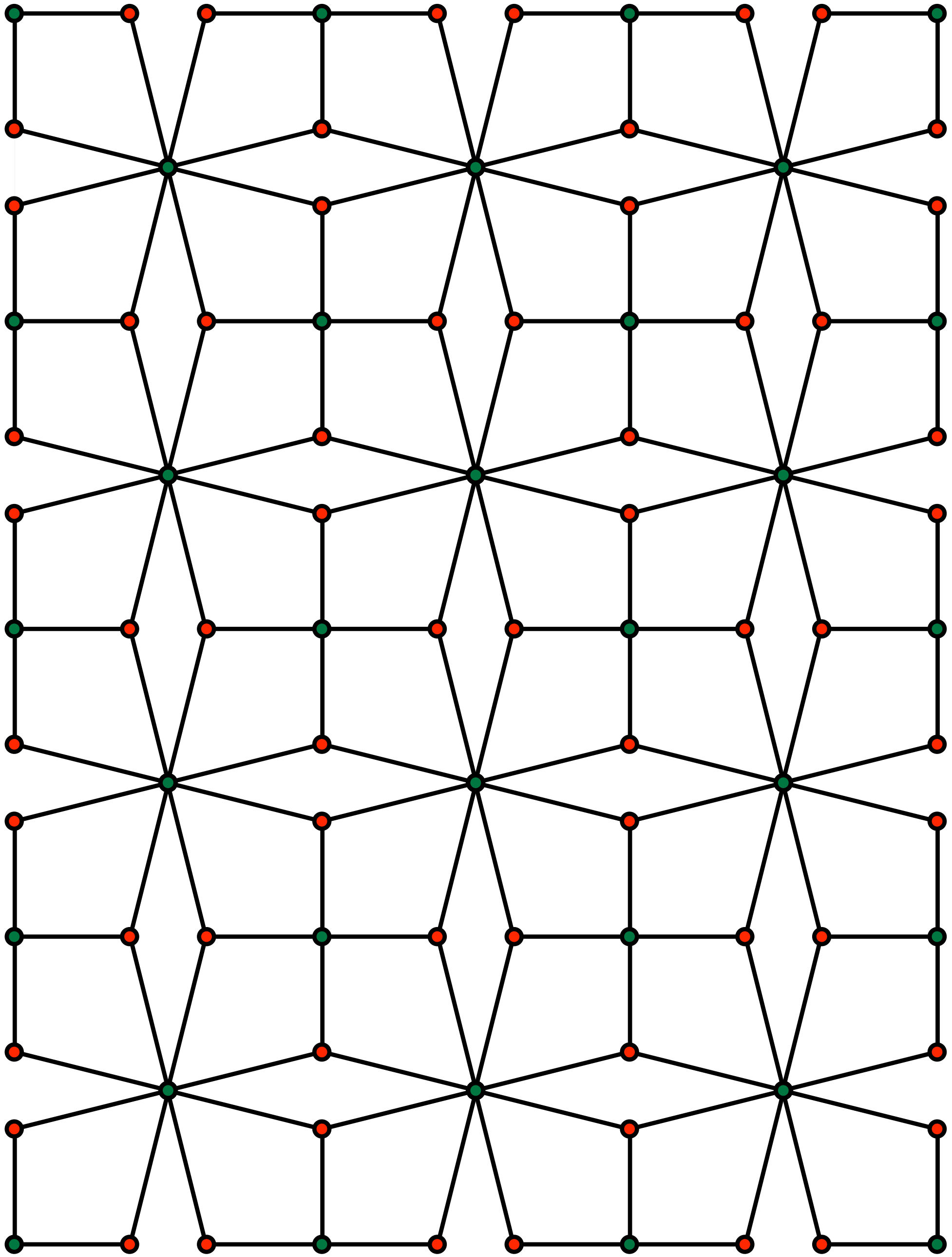
POTTS AF

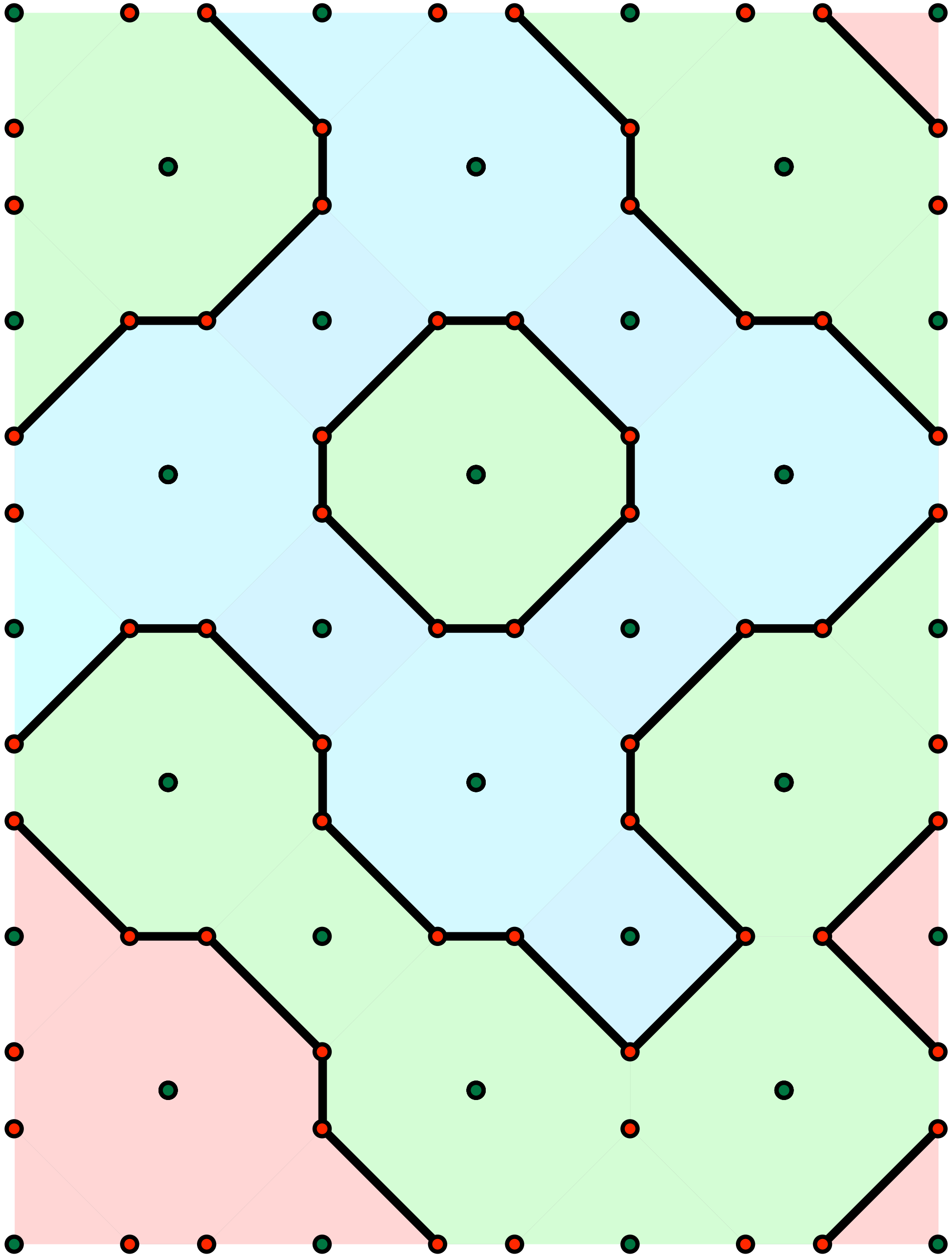
R. KOTECKÝ, C. MOORE

Abstract. Potts AF

Keywords: Potts AF







Phase Transition in the Three-State Potts Antiferromagnet on the Diced Lattice

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[1]*Center for Theoretical Study, Charles University, Prague, Czech Republic*
[2]*Mathematics Institute, University of Warwick, Coventry CV4 7AL, United Kingdom*
[3]*Gregorio Millán Institute, Universidad Carlos III de Madrid, 28911 Leganés, Spain*
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[5]*Department of Mathematics, University College London, London WC1E 6BT, United Kingdom*
(Received 18 February 2008; revised manuscript received 22 April 2008; published 17 July 2008)

We prove that the 3-state Potts antiferromagnet on the diced lattice (dual of the kagome lattice) has entropically driven long-range order at low temperatures (including zero). We then present Monte Carlo simulations, using a cluster algorithm, of the 3-state and 4-state models. The 3-state model has a phase transition to the high-temperature disordered phase at $v = e^J - 1 = -0.860\,599 \pm 0.000\,004$ that appears to be in the universality class of the 3-state Potts ferromagnet. The 4-state model is disordered throughout the physical region, including at zero temperature.

DOI: 10.1103/PhysRevLett.101.030601 PACS numbers: 64.60.Cn, 05.50.+q, 11.10.Kk, 64.60.F−

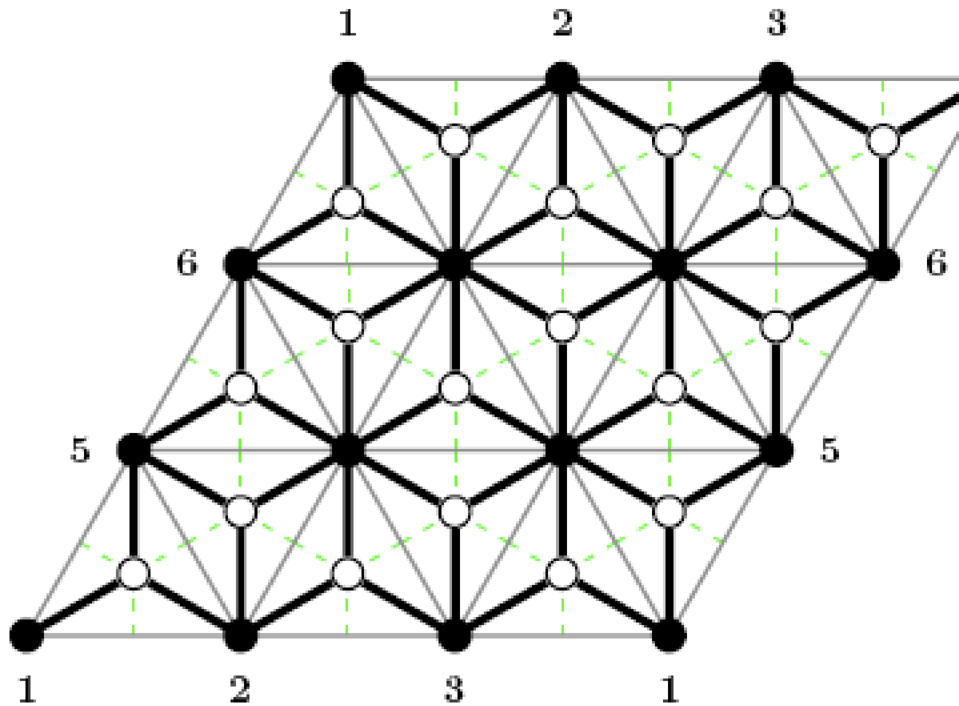
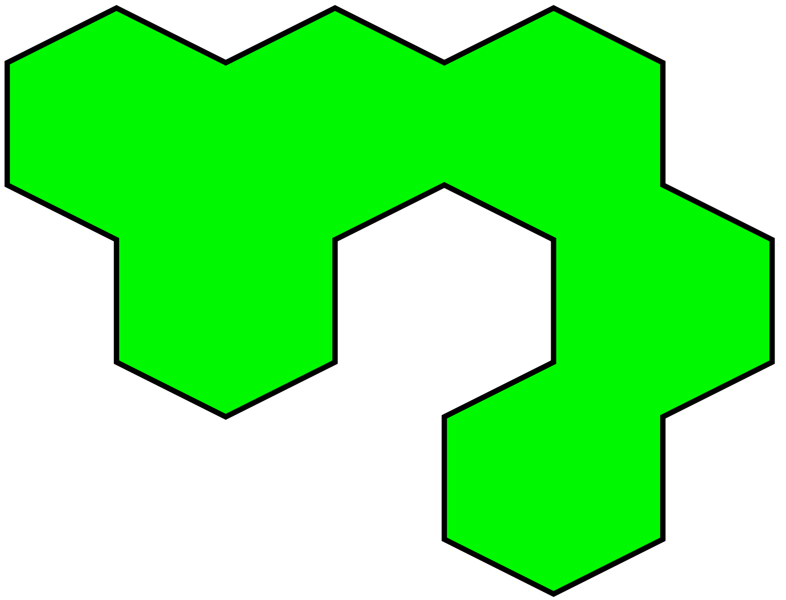


Figure 1: A diced lattice of size 3×3 with periodic boundary conditions (edges depicted with thick black lines). The full circles show the sites of degree 6, which form a triangular lattice (edges depicted with thin gray lines). The open circles show the sites of degree 3, which form a hexagonal lattice (edges depicted with thin dashed green lines) that is the dual of the triangular lattice. Periodic boundary conditions are implemented by identifying border sites with the same label.



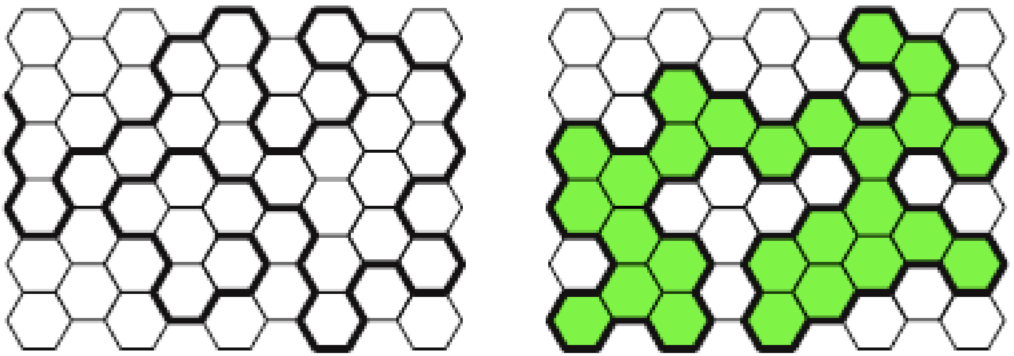
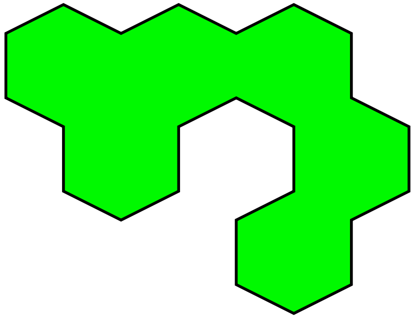


Figure 1. Examples of a self-avoiding walk (left panel) and polygon (right panel) on the honeycomb lattice.

$$2\sum_{n=6}^{\infty} q_n (1/2)^n < 2/3$$



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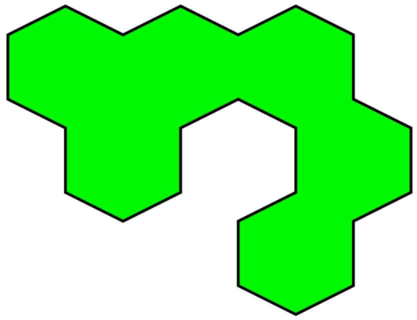
q_n is the number of n-step hexagonal SAPs surrounding the origin



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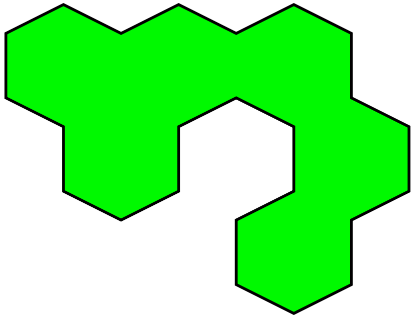
$$q_n < (n^2/36)1.868832^n$$



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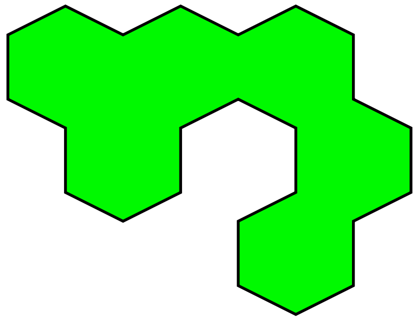

NOT ENOUGH

Institute of Physics Publishing
doi:10.1088/1742-6596/42/1/016

Journal of Physics: Conference Series 42 (2006) 163–178
Counting Complexity

Honeycomb lattice polygons and walks as a test of series analysis techniques

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Abstract. We have calculated long series expansions for self-avoiding walks and polygons on the honeycomb lattice, including series for metric properties such as mean-squared radius of gyration as well as series for moments of the area-distribution for polygons. Analysis of the series yields accurate estimates for the connective constant, critical exponents and amplitudes of honeycomb self-avoiding walks and polygons. The results from the numerical analysis agree to a high degree of accuracy with theoretical predictions for these quantities.

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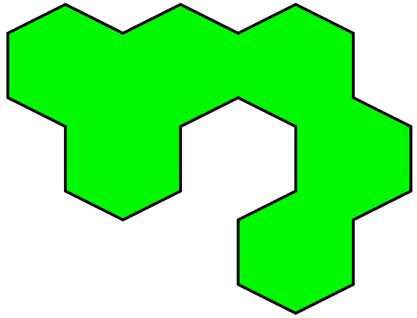

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$$2\sum_{n=6}^{\infty} q_n(1/2)^n < 2/3$$

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$$q_n < (n^2/36)1.868832^n$$

NOT ENOUGH

$$2\sum_{n=6}^{\infty} q_n (1/2)^n < 2/3$$

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$$q_n < (n^2/36)1.868832^n$$

NOT ENOUGH

$$q_n < (n^2/36)1.868832^{n-2}$$

$$2\sum_{n=6}^{\infty} q_n (1/2)^n < 2/3$$

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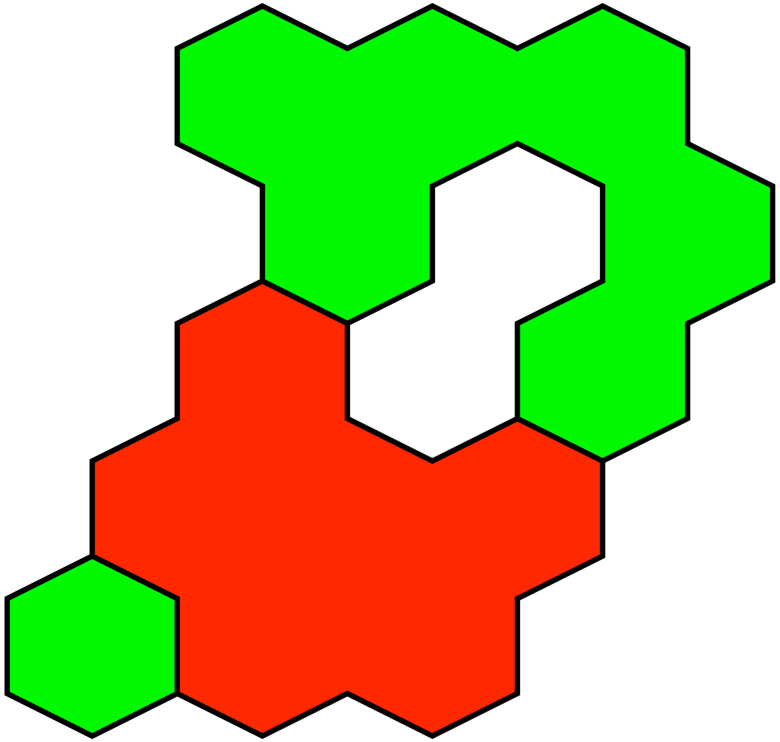
$$1/2 < 0.503417$$



Gibbs Measures and Dismantlable Graphs

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We model physical systems with "hard constraints" by the space $\mathrm{Hom}(G, H)$ of homomorphisms from a locally finite graph G to a fixed finite constraint graph H. Two homomorphisms are deemed to be adjacent if they differ on a single site of G. We investigate what appears to be a fundamental dichotomy of constraint graphs, by giving various characterizations of a class of graphs that we call *dismantlable*. For instance, H is dismantlable if and only if, for every G, any two homomorphisms from G to H which differ at only finitely many sites are joined by a path in $\mathrm{Hom}(G, H)$. If H is dismantlable, then, for any G of bounded degree, there is some assignment of activities to the nodes of H for which there is a unique Gibbs measure on $\mathrm{Hom}(G, H)$. On the other hand, if H is not dismantlable (and not too trivial), then there is some r such that, whatever the assignment of activities on H, there are uncountably many Gibbs measures on $\mathrm{Hom}(\mathsf{T}_r, H)$, where T_r is the $(r+1)$-regular tree.

$x, y \geq 0$ and $\gamma \in \mathcal{C} \cup (\cup_{i \in \mathbb{H}} \mathcal{G}^{(i)})$ ... $w(\gamma) = x^{|\gamma|} y^{t(\gamma)}$
$|\gamma|$ is the number of edges in γ and
$t(\gamma)$ is the number of 3-degree vertices in γ

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Suppose that there exist constants M and $\bar{\mu}$ such that (for any $i \in \mathbb{H}$)

$$c_n \ \equiv \ \#\{G \in \mathcal{G}_0^{(i)} : |G| = n\} \leq \bar{M}\,\bar{\mu}^n. \tag{1}$$

c_n is the number of n-step SAWs on the hexagonal lattice starting at a specified vertex and ending anywhere

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$x \in [0, 1/\bar{\mu})$ ... $K(x) \;=\; \sum_{n=0}^{\infty} c_n x^n$

$$K(x) \;\leq\; \frac{\bar{M}}{1 - x\bar{\mu}} \;<\; \infty\,.$$

Proposition 1. *Suppose that $x, y \geq 0$ satisfy $x\bar{\mu} < 1$, $xyK(x) < 1$ and $4yK(x)^2 \leq [1 - xyK(x)]^2$; and let us define*

$$L(x,y) \;=\; \frac{2K(x)}{1 - xyK(x) + \sqrt{[1 - xyK(x))]^2 - 4yK(x)^2}}\;. \tag{3}$$

Then

$$\sum_{G \in \mathcal{G}^{(i)}} w(G) \;\leq\; L(x,y) \tag{4}$$

and, for any $\gamma_0 \in \mathcal{C}_0$,

$$\sum_{\gamma \in \mathcal{C} : S(\gamma) = \gamma_0} w(\gamma) \;\leq\; w(\gamma_0)\,[1 + yL(x,y)]^{|\gamma_0|/2}. \tag{5}$$

Proposition 1. *Suppose that* $x, y \geq 0$ *satisfy* $x\bar{\mu} < 1$, $xyK(x) < 1$ *and* $4yK(x)^2 \leq [1 - xyK(x)]^2$; *and let us define*

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