Computer solution of fluid-structure vibrations on unbounded domains*

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Abstract. The aim of this research is to obtain an efficient code to compute the steady state vibrations of an elastic object submerged into a compressible fluid, in case that the fluid domain is unbounded. An example of application is the computation of scattered acoustic waves typically used for underwater detection by means of sonar (see for instance [1]). The submerged object is assumed to be linearly elastic and is described by means of its displacement field, discretized by piecewise linear continuous elements. The fluid is supposed inviscid, barotropic and compressible; it is described by means of a scalar displacement potential discretized with the same type of elements. The radiation at infinity, as given by Sommerset condition, is taken into account by means of a *DtN* (Dirichlet-to-Neumann) operator. To do this, an artificial circular boundary containing the whole elastic object is introduced and the *DtN* operator is approximated by using a truncated Fourier expansion in the eigenmodes of the exterior problem (see for instance [2]). Numerical results validating the developed code are presented.

Keywords— Fluid-structure interaction, Helmholtz problem, coupled DtN/FEM methods.

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A parallel domain decomposition scheme for Fredholm integral equations*

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Abstract. We use a domain decomposition technique to show that the solution of some Fredholm integral equations of the second kind can be obtained by solving a system of equations, of the same type, but defined on subdomains. In order to solve the system we use a parallelizable iterative scheme and we give a suficient condition for its convergence. The system is discretized by Nyström method and convergence of the numerical procedure is also considered. Finally, we apply the corresponding parallel scheme to some model problems and show the performance results obtained on a parallel computer Origin 2000 with 6 processors.

Keywords— Parallel Processing, Fredholm Integral Equations, Nyström Methods, Domain Decomposition Methods.

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Nyström and Galerkin separation techniques for Fredholm integral equations of the second kind*

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Abstract. We present the Domain Separation technique as a procedure that permits to work on subsets of the original nodes set in the discretized integral equation. In order to achieve this, it is necessary to define projection and restriction operators between the originally discretized problem and the new lower-size subproblems. The scope of this approach is to get always lower-size problems and parallelism for the resolution of the integral equations. Also, we show numerical results with Nyström quadrature and some projection operators arising in the Galerkin method for integral equations.

Keywords— Integral Equations, Nyström Method, Galerkin Method, Multigrid Techniques, Domain Reduction Method.

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On stationary model of magneto-micropolar fluid in exterior domains*

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Abstract. As in many non-bounded physical situations, the study of the dynamics of the magneto-micropolar fluid model considered on an unbounded domain plays an important and useful role. We often find solid-fluid structures in which a bounded obstacle stops or impedes the flow of the surrounding fluid and the spatial volume of the external environment of the body, namely exterior domain, is extensively much larger than the obstacle. ¿From the modeling point of view, this case may be regarded as a compact domain located in all of $I\!R^3$. Let K denote this compact subset, and let Ω denote its complement in $I\!R^3$, that is, $\Omega = K^c$. We deal with the existence of a weak solutions for equations that describe the motion of a viscous incompressible magneto-micropolar fluid in the exterior domain Ω (see [1]). Such mathematical model (e.g., see [2]) reads: Find the three-dimensional fields $(u, w, h) : \Omega \to I\!R^9$ and the scalar functions $(p, q) : \Omega \to I\!R^2$ which satisfy the system of equations:

$$-(\mu+\chi)\Delta u + (u\cdot\nabla)u + \nabla(p + \frac{r}{2}|h|^2) = \chi \operatorname{rot} w + r\,(h\cdot\nabla)h + f \quad \text{in}\,\Omega\,, \tag{1.1}$$

$$-\gamma\Delta w - (\alpha+\beta)\nabla \operatorname{div} w + j\,(u\cdot\nabla)w + 2\chi w = \chi \operatorname{rot} u + g \quad \text{in}\,\Omega\,, \tag{1.2}$$

$$-\nu\Delta h + (u\cdot\nabla)h - (h\cdot\nabla)u + \nabla q = 0 \quad \text{in}\,\Omega\,, \tag{1.3}$$

$$\operatorname{div} u = 0, \quad \operatorname{div} h = 0 \quad \text{in}\,\Omega\,, \tag{1.4}$$

$$u(x) = w(x) = h(x) = 0, \quad \text{on}\,\partial\Omega\,. \tag{1.5}$$

Here $u(x), w(x), h(x) \in I\!R^3$ denote, respectively, the velocity, the microrotational velocity, and the magnetic field of the fluid at point $x \in \Omega$, and $p(x), q(x) \in I\!R$ denote the hydrostatic and magnetic pressures at the same place. The values $\mu, \chi, r, \alpha, \beta, \gamma, j$ and ν are constants associated to properties of the material and $f(x)$, $g(x) \in I\!R^3$ are given external fields. To complete the system of equations, we establish the behaviour of the solution at infinity. We consider the classical homogeneous decay

$$\lim_{|x|\to\infty} u(x) = \lim_{|x|\to\infty} w(x) = \lim_{|x|\to\infty} h(x) = 0\,. \tag{1.6}$$

It is important to remark that our case involves solutions in the space $J_0(\Omega)$, defined as

$$J_0(\Omega) = \text{Completion of } C^\infty_{0,\sigma}(\Omega) \text{ in norm } \|\nabla\phi\|_{L^2(\Omega)}\,,$$

hence no weighted spaces are required.

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Equation (1.1) has the familiar form of the Navier-Stokes equations though coupled with (1.2) and (1.3). Equation (1.2) describes the motion inside the macro-volumes as they undergo micro rotational effects, which are represented by the micro rotational velocity vector w. For fluids with no micro structure, this velocity vanishes and we deal with a magneto-hydrodynamics system. For Newtonian fluids, where $\chi = 0$, equation (1.1) decouples from equation (1.2). Equation (1.3), which is the equation for h, is none other than the Maxwell system in which the electrical field is determined in a posteriori way. It is also important to note that if $h = 0$, we consider the well known stationary asymmetric fluid model. In this lecture, to prove existence of weak solutions we use "the extending domain method" as in Ladyzhenskaya[3] and Heywood [4]. We also discuss the uniqueness of solutions. Thus we study weak solutions the same way as in the classical study of the Navier-Stokes equations. Finally, we propose a scheme to carry out the numerical experience in the bounded domain case (see [5]).

Keywords— Magneto-Micropolar Fluids, Exterior Domains, Non-linear EDP.

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Domain decomposition methods for exterior boundary value problems in potential theory*

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Abstract. In this paper we present new domain decomposition schemes for a class of linear and nonlinear boundary value problems from potential theory. We first use the boundary integral equation method to derive a Dirichlet-to-Neumann mapping to transform the exterior problem into an equivalent problem on a bounded domain. This bounded domain is decomposed into a finite number of annular subdomains and different types of interface unknowns are considered. The Steklov-Poincaré operators are expressed either by the finite element method or a combination between finite element and boundary element methods. The corresponding interface problems are solved by using Robin-type preconditioners which yields iteration-by-subdomain algorithms well suited for parallel processing. Also, we present contractivity results and finite dimensional approximations. Finally, some numerical experiments on a parallel computer are also provided.

Keywords— Dirichlet-to-Neumann Mapping, Steklov-Poincaré Operator, Boundary Element Method, Finite Element Method, Domain Decomposition, Preconditioned Conjugate Gradient Method.

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Primal and dual formulations for the coupling of mixed-FEM and BEM*

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Abstract. This paper deals with the coupling of **mixed** finite element and boundary element methods. The procedure, which may also use Dirichlet-to-Neumann mappings, is applied to several transmission problems in physics and engineering sciences, particularly potential theory and elastostatics. Our analysis includes both primal and dual mixed methods, and consider linear and nonlinear boundary value problems. We study the different types of variational formulations arising from the coupling procedure and derive results of existence and uniqueness of solution for the continuous and discrete schemes. A-priori and a-posteriori estimates for the corresponding error are also provided.

Keywords— Mixed Finite Elements, Boundary Integral Equation Method, Dirichlet-to-Neumann Mappings.

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Quality improvement of Delaunay triangulations for finite element and finite volume discretization methods*

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Abstract. In this paper we discuss the application of a new longest-edge based algorithm capable of improving the quality of Delaunay triangulations for finite element and finite volume discretization methods. The resulting triangulations are Delaunay triangulations, whose boundary triangles (triangles with at least one edge on the boundary or on a material interface) do not have obtuse angles opposite to any boundary or interface edges. In addition, the algorithm guarantees that, for non geometry restricted triangles, the minimum and maximum angles of the triangles are bounded (minimum angle greater than or equal to 30° and maximum angle less than or equal to 120°).

Keywords— Longest-Edge, Delaunay Triangulations, FEM, Finite Volume Methods.

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Numerical comparison between relaxation and nonlinear equilibrium models. Application to chemical engineering*

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Abstract. A model to take into account the finite exchange kinetic for the equilibrium between phases in the chromatography experiment and the distillation is studied. The resulting hyperbolic system with a nonlinear relaxation term is then formally treated with a Chapman-Enskog type expansion. A first order correction to the classical quasilinear hyperbolic model is derived which consists in a nonlinear diffusion term. Numerical schemes for both models– relaxed and parabolic – are then tested and compared for different initial and boundary values. A range of validity for the first order approximation is obtained through numerical simulations where its computational performance indicates that it should be advantageously used.

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The aliasing error in wavelet boundary element methods*

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Abstract. We first observe that the wavelet boundary element method (WBEM), as presented in [1], consists in considering a wavelet transform in which the coefficients are obtained by sampling instead of numerical quadrature. In this paper we use the Sampling Theorem for Wavelet subspaces from [2], to determine the *aliasing* error in the multiscale approximation. In other words, we find an upper bound for the L^2-error between a function $f \in L^2[0, L]$, which is L-periodic, Hölder continuous in $(0, L)$ and unbounded at $x = 0$, and the interpolation function constructed in a subspace of fixed resolution. In addition, we show that this interpolation function reduces to a linear combination of the translates of the scale function of that given subspace. The corresponding coefficients are obtained multiplying a circulant matrix by a vector containing the sampling values of the function, to which the *singularity sustraction method* has been previously applied. Some numerical examples are also provided.

Keywords— Wavelets, Reproducing Kernel, Riesz Basis, Zak Transform, Circulant Matrix, Aliasing Error, Integral Equation, Boundary Element.

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Longest-edge refinement algorithms in 3-dimensions*

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Abstract. Longest-edge algorithms are well known techniques for the refinement of triangular 2-dimensional meshes, which guarantee strong non degeneracy properties over the refined meshes: they produce triangles whose minimum angle is bounded as a function of the quality of the initial triangulation, they produce a finite number of similarly distinct triangles, a fractal behavior around each vertex holds, and both the percentage of good-quality triangles and the area covered by these triangles increase as the refinement proceeds. In this paper, some mesh non-degeneracy properties for longest-edge 3-dimensional algorithms are discussed. Empirical experimentation showing analogous behavior to the 2-dimensional case is also presented.

Keywords— Longest-Edge Algorithms, Mesh Refinements, Non-Degeneracy.

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Modeling and numerical solution of elastoacoustic vibrations with interface damping*

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Abstract. This paper deals with the damped transmission of small amplitude vibrations between two media: a linear elastic structure and an inviscid, compressible, barotropic fluid, both separated by a thin layer of viscoelastic material with a given frequency dependent impedance. Much attention has been recently paid to this kind of problems, mostly related to the goal of decreasing the level of noise in aircrafts and cars (see for instance [4]). We consider a displacement formulation for both, the fluid and the solid, which has been thoroughly analized in the case of an undamped interaction in some previous papers ([1], [2], [3]). Standard finite elements for the structure combined with Raviart-Thomas elements for the fluid has been shown therein to provide a reliable method to compute the vibration modes of the coupled system. In particular, the method was proved to be free of spurious non-zero frequency modes. In this paper we present a simplified mathematical model for the damped transmission conditions which allows us to introduce an efficient numerical method to compute the vibrations of the coupled system. It consists of relaxing the kinematic constrain: the classical slipping condition on the interface is substituted by

$$p = i\omega Z(\omega)(u^S \cdot n - u^F \cdot n),$$

where p is the pressure on the interface, ω the excitation frequence, $Z(\omega)$ the (complex) frequency dependent acoustic impedance of the viscoelastic material and $u^S \cdot n$ and $u^F \cdot n$ the normal displacements of the solid and the fluid, respectively. The method is used to compute the vibrations transmitted to the fluid from an external source acting on the structure and the (complex) resonance frequencies of the coupled damped system. The relation between these and the response peaks of the system is also analized.

Keywords— Damped Acoustic Vibrations, Quadratic Eigenvalue Problems.

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