

Index

- $n!$, 80
 π , estimation of, 43–46
absorbing Markov chain, 415
absorbing state, 416
AbsorbingChain (program), 421
absorption probabilities, 420
Ace, Mr., 241
Ali, 178
alleles, 348
AllPermutations (program), 84
ANDERSON, C. L., 157
annuity, 246
 life, 246
 terminal, 246
arc sine laws, 493
area, estimation of, 42
Areabargraph (program), 46
asymptotically equal, 81

Baba, 178
babies, 14, 250
Banach’s Matchbox, 254
BAR-HILLEL, M., 176
BARNES, B., 175
BARNHART, R., 11
BAYER, D., 119
Bayes (program), 147
Bayes probability, 136
Bayes’ formula, 146
BAYES, T., 149
beard, 153
bell-shaped, 47
Benford distribution, 195
BENKOSKI, S., 40
Bernoulli trials process, 95
BERNOULLI, D., 227

BERNOULLI, J., 112, 149, 310–312
Bertrand’s paradox, 47–50
BERTRAND, J., 49, 181
BertrandsParadox (program), 49
beta density, 168
BIENAYMÉ, I., 310, 378
BIGGS, N. L., 85
binary expansion, 69
binomial coefficient, 92
binomial distribution, 98, 184
 approximating a, 329
Binomial Theorem, 102
BinomialPlot (program), 98
BinomialProbabilities (program), 98
Birthday (program), 78
birthday problem, 77
blackjack, 247, 253
blood test, 254
Bose-Einstein statistics, 106
Box paradox, 181
BOX, G. E. P., 213
boxcars, 27
BRAMS, S., 179, 182
Branch (program), 382
branching process, 377
 customer, 394
BranchingSimulation (program), 388
bridge, 181, 182, 199, 203, 287
BROWN, B. H., 38
BROWN, E., 425
Buffon’s needle, 44–46, 51–53
BUFFON, G. L., 9, 44, 50–51
BuffonsNeedle (program), 45
bus paradox, 164

calendar, 38
cancer, 147

canonical form of an absorbing Markov chain, 417
 car, 137
CARDANO, G., 30–31, 111, 249
 cars on a highway, 66
CASANOVA, G., 11
 Cauchy density, 218, 401
 cells, 347
 Central Limit Theorem, 325
 for Bernoulli Trials, 330
 for Binomial Distributions, 328
 for continuous independent trials process, 357
 for discrete independent random variables, 345
 for discrete independent trials process, 343
 for Markov Chains, 464
 proof of, 398
 chain letter, 389
 characteristic function, 398
 Chebyshev Inequality, 305, 316
CHEBYSHEV, P. L., 313
 chi-squared density, 216, 296
 Chicago World’s Fair, 52
 chord, random, 47, 54
 chromosomes, 348
CHU, S.-C., 109
CHUNG, K. L., 153
 Circle of Gold, 389
 Clinton, Bill, 196
 clover-leaf interchange, 39
CLTBernoulliGlobal, 332
CLTBernoulliLocal (program), 329
CLTBernoulliPlot (program), 327
CLTGeneral (program), 345
CLTIndTrialsLocal (program), 342
CLTIndTrialsPlot (program), 341
COATES, R. M., 305
CoinTosses (program), 3
 Collins, People v., 153, 202
 color-blindness, 424
 conditional density, 162
 conditional distribution, 134
 conditional expectation, 239

conditional probability, 133
CONDORCET, Le Marquis de, 12
 confidence interval, 334, 360
 conjunction fallacy, 38
 continuum, 41
 convolution, 286, 291
 of binomial distributions, 289
 of Cauchy densities, 294
 of exponential densities, 292, 300
 of geometric distributions, 289
 of normal densities, 294
 of standard normal densities, 299
 of uniform densities, 292, 299
CONWAY, J., 432
CRAMER, G., 227
 craps, 235, 468
 Craps (program), 235
CROSSEN, C., 161
CROWELL, R., 468
 cumulative distribution function, 61
 joint, 165
 customer branching process, 394
 cut, 119
 Dartmouth, 27
 darts, 56, 57, 59, 60, 64, 71, 163, 164
 Darts (program), 58
DAVID, F. N., 86, 337, 489
DAVID, F. N., 32
de MOIVRE, A., 37, 87, 148, 336, 489
de MONTMORT, P. R., 85
de MÉRÉ, CHEVALIER, 4, 31, 37
 degrees of freedom, 217
DeMere1 (program), 4
DeMere2 (program), 4
 density function, 56, 59
 beta, 168
 Cauchy, 218, 401
 chi-squared, 216, 296
 conditional, 162
 exponential, 53, 66, 163, 205
 gamma, 207
 joint, 165
 log normal, 224
 Maxwell, 215, 295

- normal, 212
t-, 360
uniform, 60, 205
derangement, 85
DIACONIS, P., 119, 251
Die (program), 225
DieTest (program), 297
distribution function, 1, 19
 properties of, 22
 Benford, 195
 binomial, 184
 geometric, 184
 hypergeometric, 193
 joint, 142
 marginal, 143
 negative binomial, 186
 Poisson, 187
 uniform, 183
DNA, 348
DOEBLIN, W., 449
DOYLE, P. G., 87, 470
Drunkard's Walk example, 416, 419–
 421, 423, 427, 443
Dry Gulch, 279

EDWARDS, A. W. F., 107
Egypt, 30
Ehrenfest model, 410, 433, 441, 460,
 461
EHRENFEST, P., 410
EHRENFEST, T., 410
EhrenfestUrn (program), 462
EISENBERG, B., 160
elevator, 88, 115
Emile's restaurant, 75
ENGLE, A., 445
envelopes, 179, 180
EPSTEIN, R., 287
equalization, 472
equalizations
 expected number of, 479
ergodic Markov chain, 433
ESP, 250, 251
EUCLID, 85
Euler's formula, 202
Eulerian number, 126
event, 18
events
 attraction of, 160
 independent, 139, 164
 repulsion of, 160
existence of God, 245
expected value, 226, 268
exponential density, 53, 66, 163, 205
extinction, problem of, 379

factorial, 80
fair game, 241
FALK, R., 161, 176
fall, 130
fallacy, 38
FELLER, W., 11, 106, 191, 201, 218,
 254, 344
FERMAT, P., 4, 32–35, 111–112, 156
Fermi-Dirac statistics, 106
figurate numbers, 107
financial records
 suspicious, 196
finite additivity property, 23
FINN, J., 178
First Fundamental Mystery of Probability, 232
first maximum of a random walk, 496
first return to the origin, 473
Fisher's Exact Test, 193
FISHER, R. A., 252
fixed column vector, 435
fixed points, 82
fixed row vector, 435
FixedPoints (program), 82
FixedVector (program), 437
flying bombs, 191, 201
Fourier transform, 398
FRECHET, M., 466
frequency concept of probability, 70
frustration solitaire, 86
Fundamental Limit Theorem for Regular Markov Chains, 448
fundamental matrix, 419
 for a regular Markov chain, 457

- for an ergodic Markov chain, 458
- GALAMBOS, J., 303
- GALILEO, G., 12
- Gallup Poll, 14, 336
- Galton board, 100, 351
- GALTON, F., 281, 345, 350, 377
- GaltonBoard (program), 100
- Gambler's Ruin, 426, 486, 487
- gambling systems, 241
- gamma density, 207
- GARDNER, M., 181
- gas diffusion
 - Ehrenfest model of, 410, 433, 441, 460, 461
- GELLER, S., 176
- GeneralSimulation (program), 9
- generating function
 - for continuous density, 394
 - moment, 366, 395
 - ordinary, 370
- genes, 348, 411
- genetics, 345
- genotypes, 348
- geometric distribution, 184
- geometric series, 29
- GHOSH, B. K., 160
- goat, 137
- GONSHOR, H., 425
- GOSSET, W. S., 360
- grade point average, 343
- GRAHAM, R., 251
- GRANBERG, D., 161
- GRAUNT, J., 246
- Greece, 30
- GRIDGEMAN, N. T., 51, 181
- GRINSTEAD, C. M., 87
- GUDDER, S., 160
- HACKING, I., 30, 148
- HAMMING, R. W., 283, 284
- HANES data, 345
- Hangtown, 279
- Hanover Inn, 65
- hard drive, Warp 9, 66
- Hardy-Weinberg Law, 349
- harmonic function, 428
- Harvard, 27
- hat check problem, 82, 85, 104
- heights
 - distribution of, 345
- helium, 106
- HEYDE, C., 378
- HILL, T., 196
- Holmes, Sherlock, 91
- HorseRace (program), 6
- hospital, 14, 250
- HOWARD, R. A., 406
- HTSimulation (program), 6
- HUDDE, J., 148
- HUIZINGA, F., 389
- HUYGENS, C., 147, 243–245
- hypergeometric distribution, 193
- hypotheses, 145
- hypothesis testing, 100
- Inclusion-Exclusion Principle, 103
- independence of events, 139, 164
 - mutual, 141
- independence of random variables
 - mutual, 143, 165
- independence of random
 - variables, 143, 165
- independent trials process, 144, 168
- interarrival time, average, 208
- interleaving, 119
- irreducible Markov chain, 433
- Isle Royale, 202
- JAYNES, E. T., 49
- JOHNSONBOOUGH, R., 153
- joint cumulative distribution
 - function, 165
- joint density function, 165
- joint distribution function, 142
- joint random variable, 142
- KAHNEMAN, D., 38
- Kemeny's constant, 469, 470
- KEMENY, J. G., 200, 406, 466
- KENDALL, D. G., 378

- KEYFITZ, N., 383
KILGOUR, D. M., 179, 182
KINGSTON, J. G., 157
KONOLD, C., 161
KOZELKA, R. M., 344

Labouchere betting system, 12, 13
LABOUCHERE, H. du P., 12
LAMPERTI, J., 267, 324
LAPLACE, P. S., 51, 53, 350
last return to the origin, 482
Law (program), 310
Law of Averages, 70
Law of Large Numbers, 307, 316
 for Ergodic Markov Chains, 439
 Strong, 70
LawContinuous (program), 318
lead change, 482
LEONARD, B., 256
LEONTIEF, W. W., 426
LEVASSEUR, K., 485
library problem, 82
life table, 39
light bulb, 66, 72, 172
LINDEBERG, J. W., 344
LIPSON, A., 161
Little's law for queues, 276
Lockhorn, Mr. and Mrs., 65
log normal density, 224
lottery
 Powerball, 204
LUCAS, E., 118

MAISTROV, L., 150, 310
MANN, B., 119
margin of error, 335
marginal distribution function, 143
Markov chain, 405
 absorbing, 415
 ergodic, 433
 irreducible, 433
 regular, 433
Markov Chains
 Central Limit Theorem for, 464
 Fundamental Limit Theorem for
 Regular, 448

MARKOV, A. A., 464
martingale, 241, 242, 428
 origin of word, 11
martingale betting system, 11, 14, 248
matrix
 fundamental, 419
MatrixPowers (program), 407
maximum likelihood
 estimate, 198, 202
Maximum Likelihood
 Principle, 91, 116
Maxwell density, 215, 295
maze, 440, 453
McCRACKEN, D., 10
mean, 226
mean first passage matrix, 455
mean first passage time, 452
mean recurrence matrix, 455
mean recurrence time, 454
memoryless property, 68, 164, 206
milk, 252
modular arithmetic, 10
moment generating function, 366, 395
moment problem, 368, 398
moments, 365, 394
Monopoly, 469
MonteCarlo (program), 42
Monty Hall problem, 136, 161
moose, 202
mortality table, 246
mule kicks, 201
MULLER, M. E., 213
multiple-gene hypothesis, 348
mustache, 153
mutually independent events, 141
mutually independent random
 variables, 143

negative binomial distribution, 186
NEGRINI, M., 196
New York Times, 340
New York Yankees, 117, 253
New-Age Solitaire, 129
NEWCOMB, S., 196
NFoldConvolution (program), 287

- normal density, 47, 212
- NormalArea (program), 322
- nursery rhyme, 84
- odds, 27
- ordering, random, 126
- ordinary generating function, 370
- ORE, O., 30, 31
- outcome, 18
- Oz, Land of, 406, 439
- Pascal's triangle, 93, 103, 107
- PASCAL, B., 4, 32–35, 107, 111–112, 156, 242, 245
- paternity suit, 222
- PEARSON, K., 9, 351
- PENNEY, W., 432
- People v. Collins, 153, 202
- PERLMAN, M. D., 45
- permutation, 79
 - fixed points of, 82
- Philadelphia 76ers, 15
- photons, 106
- Pickwick, Mr., 153
- Pilsdorff Beer Company, 279
- PITTEL, B., 255
- point count, 287
- Poisson approximation to the binomial distribution, 189
- Poisson distribution, 187
 - variance of, 262
- poker, 95
- polls, 333
- Polya urn model, 152, 174
- ponytail, 153
- posterior probabilities, 145
- Powerball lottery, 204
- PowerCurve (program), 102
- Presidential election, 336
- PRICE, C., 86
- prior probabilities, 145
- probability
 - Bayes, 136
 - conditional, 133
 - frequency concept of, 2
- of an event, 19
- transition, 406
- vector, 407
- problem of points, 32, 111, 147, 156
- process, random, 127
- PROPP, J., 256
- PROSSER, R., 200
- protons, 106
- PÓLYA, G., 15, 17, 475
- quadratic equation, roots of, 73
- quantum mechanics, 106
- QUETELET, A., 350
- Queue (program), 208
- queues, 186, 208, 275
- quincunx, 351
- RABELAIS, F., 12
- racquetball, 157
- radioactive isotope, 66, 71
- RAND Corporation, 10
- random integer, 39
- random number generator, 2
- random ordering, 126
- random process, 127
- random variable, 1, 18
 - continuous, 58
 - discrete, 18
 - functions of a, 210
 - joint, 142
- random variables
 - independence of, 143
 - mutual independence of, 143
- random walk, 471
 - in n dimensions, 17
- RandomNumbers (program), 3
- RandomPermutation (program), 82
- rank event, 160
- raquetball, 13
- rat, 440, 453
- records, 83, 234
- Records (program), 84
- regression on the mean, 282
- regression to the mean, 345, 352
- regular Markov chain, 433

- reliability of a system, 154
restricted choice, principle of, 182
return to the origin, 472
 first, 473
 last, 482
 probability of eventual, 475
reversibility, 463
reversion, 352
riffle shuffle, 119
RIORDAN, J., 86
rising sequence, 120
rnd, 42
ROBERTS, F., 426
Rome, 30
ROSS, S., 270, 276
roulette, 13, 237, 432
run, 229
RÉNYI, A., 167
- SAGAN, H., 237
sample, 333
sample mean, 265
sample space, 18
 continuous, 58
 countably infinite, 28
 infinite, 28
sample standard deviation, 265
sample variance, 265
SAWYER, S., 412
SCHULTZ, H., 254
SENETA, E., 378, 444
service time, average, 208
SHANNON, C. E., 465
SHOLANDER, M., 39
shuffling, 119
SHULTZ, H., 256
SimulateChain (program), 439
simulating a random variable, 211
snakeeyes, 27
SNELL, J. L., 87, 175, 406, 466
snowfall in Hanover, 83
spike graph, 6
Spikegraph (program), 6
spinner, 41, 55, 59, 162
spread, 266
- St. Ives, 84
St. Petersburg Paradox, 227
standard deviation, 257
standard normal random variable, 213
standardized random variable, 264
standardized sum, 326
state
 absorbing, 416
 of a Markov chain, 405
 transient, 416
statistics
 applications of the Central Limit Theorem to, 333
stepping stones, 412
SteppingStone (program), 412
stick of unit length, 73
STIFEL, M., 111
STIGLER, S., 350
Stirling's formula, 81
STIRLING, J., 87
StirlingApproximations (program), 81
stock prices, 241
StockSystem (program), 241
Strong Law of Large Numbers, 70, 314
suit event, 160
SUTHERLAND, E., 182
- t-density, 360
TARTAGLIA, N., 109
tax returns, 196
tea, 252
telephone books, 255
tennis, 157, 424
tetrahedral numbers, 107
THACKERAY, W. M., 14
THOMPSON, G. L., 406
THORP, E., 247, 253
time to absorption, 419
TIPPETT, L. H. C., 10
traits, independence of, 216
transient state, 416
transition matrix, 406

- transition probability, 406
- tree diagram, 24, 76
 - infinite binary, 69
- Treize, 85
- triangle
 - acute, 73
- triangular numbers, 107
- trout, 198
- true-false exam, 267
- Tunbridge, 154
- TVERSKY, A., 14, 38
- Two aces problem, 181
- two-armed bandit, 170
- TwoArm (program), 171
- type 1 error, 101
- type 2 error, 101
- typesetter, 189
- ULAM, S., 11
- unbiased estimator, 266
- uniform density, 205
- uniform density function, 60
- uniform distribution, 25, 183
- uniform random variables
 - sum of two continuous, 63
- unshuffle, 122
- USPENSKY, J. B., 299
- utility function, 227
- VANDERBEI, R., 175
- Vandermonde determinant, 369
- variance, 257, 271
 - calculation of, 258
- variation distance, 127
- VariationList (program), 127
- volleyball, 158
- von BORTKIEWICZ, L., 201
- von MISES, R., 87
- von NEUMANN, J., 10, 11
- vos SAVANT, M., 40, 86, 136, 176, 181
- Wall Street Journal, 161
- watches, counterfeit, 91
- WATSON, H. W., 378
- WEAVER, W., 465
- Weierstrass Approximation Theorem, 315
- WELDON, W. F. R., 9
- Wheaties, 117, 253
- WHITAKER, C., 136
- WHITEHEAD, J. H. C., 181
- WICHURA, M. J., 45
- WILF, H. S., 90, 474
- WOLF, R., 9
- WOLFORD, G., 159
- Woodstock, 154
- Yang, 129
- Yin, 129
- ZAGIER, D., 485
- Zorg, planet of, 90