Resumen MA26B

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1 Formulas de Frenet

$$\frac{d\vec{T}}{ds} = k\vec{N} \tag{1}$$

$$\frac{d\vec{N}}{ds} = -k\vec{T} + \tau\vec{B} \tag{2}$$

$$\frac{d\vec{B}}{ds} = -\tau\vec{N} \tag{3}$$

2 Coordenadas curvilineas ortogonales

$$\begin{aligned} x &= \varphi_1(u_1,u_2,u_3) \\ y &= \varphi_1(u_1,u_2,u_3) \qquad \vec{x} = \varphi(\vec{u}) \\ z &= \varphi_1(u_1,u_2,u_3) \end{aligned}$$

$$h_i = \|\frac{\partial\varphi}{\partial u_i}\| \quad \hat{u}_i = \frac{1}{h_i}\frac{\partial\varphi}{\partial u_i} \quad J_\varphi = h_1h_2h_3$$

$\{\hat{u}_1,\hat{u}_2,\hat{u}_3\}$ base ortonormal derecha

$$\begin{aligned} \nabla\phi &= \frac{1}{h_1}\frac{\partial\phi}{\partial u_1}\hat{u}_1 + \frac{1}{h_2}\frac{\partial\phi}{\partial u_2}\hat{u}_2 + \frac{1}{h_3}\frac{\partial\phi}{\partial u_3}\hat{u}_3 \\ \Delta\phi &= \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}\left(\frac{h_2h_3}{h_1}\frac{\partial\phi}{\partial u_1}\right) + \frac{\partial}{\partial u_2}\left(\frac{h_1h_3}{h_2}\frac{\partial\phi}{\partial u_2}\right) + \frac{\partial}{\partial u_3}\left(\frac{h_1h_2}{h_3}\frac{\partial\phi}{\partial u_3}\right)\right] \end{aligned}$$

Si $\vec{F} = F_1\hat{u}_1 + F_2\hat{u}_2 + F_3\hat{u}_3$

$$\begin{aligned} \mathrm{div}\vec{F} &= \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u_1}(h_2h_3F_1) + \frac{\partial}{\partial u_2}(h_1h_3F_2) + \frac{\partial}{\partial u_3}(h_1h_2F_3)\right] \\ \mathrm{rot}\vec{F} &= \frac{1}{h_2h_3}\left(\frac{\partial}{\partial u_1}(h_3F_3) - \frac{\partial}{\partial u_3}(h_2F_2)\right)\hat{u}_1 \\ &+ \frac{1}{h_1h_3}\left(\frac{\partial}{\partial u_3}(h_1F_1) - \frac{\partial}{\partial u_1}(h_3F_3)\right)\hat{u}_2 \\ &+ \frac{1}{h_1h_2}\left(\frac{\partial}{\partial u_1}(h_2F_2) - \frac{\partial}{\partial u_2}(h_1F_1)\right)\hat{u}_3 \end{aligned}$$

3 Teorema de la divergencia

Sea $\Omega \subset \mathbb{R}^3$ un abierto acotado cuya frontera $\partial\Omega$ es una superficie regular por trozos orientada según la normal exterior.
Sea $\vec{F}\colon \Omega' \to \mathbb{R}^3$ una campo vectorial de clase $\mathcal{C}^1$ sobre el abierto $\Omega' \supset \bar{\Omega}$ entonces

$$\iint\limits_{\partial\Omega} \vec{F} \cdot d\vec{S} = \iiint\limits_{\Omega} \operatorname{div}(\vec{F}) dV$$