Optimizing Open Pit Block Scheduling with Exposed Ore Reserve[1]

José Saavedra-Rosas[a], Enrique Jélvez[b], Jorge Amaya[c*] and Nelson Morales[b]

[a]*Department of Mineral and Energy Economics, Curtin University, WA, Australia, 6000*
[b]*DELPHOS Mine Planning Laboratory, Advanced Mining Technology Center, Department of Mine Engineering, Universidad de Chile, Santiago, Chile*
[c]*Center for Mathematical Modeling & Department of Mathematical Engineering, Universidad de Chile, Santiago, Chile*

* Corresponding author: Jorge Amaya, jamaya@dim.uchile.cl

Abstract

A crucial problem in the mining industry is to determine the *optimal sequence of extraction* of blocks, in which the mine has been structured for exploitation. Typically a mine can be constituted by several thousands or millions of blocks. The sequencing models for this structure are very complex giving rise to very large combinatorial linear models. Operational mine plans are usually produced on a yearly basis and further scheduling is attempted to provide monthly, weekly and daily schedules. A portion of the ore reserve is said to be exposed if it is readily available for extraction at the start of the next period. In this paper, an integer programming model is presented to generate pit designs under exposed ore reserve requirements, as an extension of the classical optimization models for mine planning. For this purpose, we introduce a set of new binary variables, representing the extraction, wasting and processing decisions. The model has been coded and tested in a set of standard instances, showing very encouraging results in the generation of operational sequences of extraction and destination of blocks.

Keywords: Mine Planning, Surface Mining, Open Pit Planning, Optimisation Model, Ore Reserve.

1. Introduction and conceptual model

Mining industry is usually a very relevant economic sector in many countries. For example in Chile, copper exploitation exports account for about 62.5% of the total exports and are responsible for 12% of Chile's GDP (Cochilco, 2013).

Open-pit mines are characterized for their high production levels and small operational costs when compared to other exploitation methods. Unfortunately, when using this exploitation technique, usually one needs to remove material with poor economic value (*waste*), to give access to more economically profitable material.

The actual value of a mining project has a clear dependency on the order in which the material is extracted and processed. Given this subdivision of time and space it is possible to define a period when a given block will be extracted which provides a *block schedule*.

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Each block is characterized by its metal content, density, lithology and other relevant attributes that are produced by using estimation techniques specifically designed to deal with the spatial nature of the mineralization. The value of a mine plan is thus determined by the value contained in the blocks that are extracted at certain periods.

Generally speaking, three different problems are usually considered by mining planners for the economic valuation, design and planning of open pit mines as pointed out by Hustrulid and Kuchta (2006). The first one is the Final Open Pit problem, also called "Ultimate Pit Limit" problem, which aims to find the region of maximal economic value for exploitation under certain geotechnical stability constraints and assuming one period of extraction and infinite capacity. For a large number of blocks, this problem can be solved through a combination between Linear Programming and some suitable heuristics. Another more realistic problem is the so-called Capacitated Final Open Pit, which considers an additional constraint on the total capacity for one-period exploitation, giving rise to nontrivial binary problems. The third problem is a multi-period version of the latter, the Capacitated Dynamic Open Pit problem, with the goal of finding an optimal sequence of extracted volumes in a certain finite time horizon for bounded capacities at each period, the optimality criterion being the total discounted profit. This is the more interesting scheduling problem for mine extraction, but here a new version of it will be introduced.

A common practice for the formulation of these problems consists in describing an ore reserve via the construction of a three-dimensional block model of the three-dimensional mining site. Each block corresponds to a unitary volume of extraction characterized by several geologic and economic properties which are estimated from sample data. Block models can be represented as directed graphs where nodes are associated with blocks, while arcs correspond to the precedence of these blocks in the ore reserve. The precedence order is induced by physical and operational constraints as those derived from the geo-mechanics of slope stability. This discrete approach gives rise to huge combinatorial problems whose mathematical formulations are special large-scale instances of Integer Programming Optimization problems, see for instance Cacetta (2007).

In open-pit mining a block schedule will be called feasible if it satisfies a set of constraints. The more important of such constrains are related to the *precedence* between blocks, as the extraction process proceeds from surface down to the bottom of the mineralization it is necessary to first remove blocks on the surface before access. This idea applies to every block in the model: it is not possible to access a given block in a certain time unless the blocks that are "above" it have been already extracted. Also, stability of the walls must be ensured which is expressed in terms of slope angles that must be satisfied at any point in time, which essentially helps to define a graph of precedence conditions. The amount of material to be transported and processed at each time period is subject to upper bounds and lower bounds given by transportation and plant capacity, respectively, which are usually expressed in tons.

The problem that this paper addresses is that of design of an exploitation stage in an open pit leaving enough ore reserve that is readily available at the start of the next extraction period. A block is said to be exposed at a given period if its precedent blocks have been all extracted but the block itself has been not. Means to identify exposed block are required and this motivates the introduction of a new type of variable with its associated constraints in a Mixed Integer Programming (MIP) model.

For the open-pit mine production scheduling problem, a very general formulation, due to Johnson (1968,1969), presents the block scheduling problem under slope, capacity and blending constraints (the last ones given by ranges of the processed ore grade) within a multi-destination setting, i.e., the optimization model decides the best process to apply on a block by block basis. Unfortunately, the complexity of the model is too big to be solved in a realistic case study which detracts from its application.

Since the introduction by Lerchs and Grossman (1965) of a model that is now considered a classic in the subject, open pit design by means of Operations Research models has reached a level of maturity and acceptance by industry becoming fundamental in every mining endeavor. They proposed a simplified version of the problem, known as the *ultimate pit* or *final pit* problem. For this simplified model the block destinations are fixed in advance and the only constraint considered is the slope constraint, hence capacity or blending constraints are not present. For this model, the problem reduces to selecting a subset of blocks containing maximal value whilst respecting the precedence constraints. The authors also presented an efficient (polynomial) algorithm for solving the ultimate pit problem, and showed that the size of the pit is monotonic on the value of the blocks, in the sense that, if the values of the blocks decrease, the set of selected blocks is a subset of the original one. These two properties are used to produce nested pits and therefore (by considering an increasing sequence of prices), using trial and error, the basic introduction of time and look for block schedules that could satisfy other constraints like capacity. Present-day commercial software, like GEOVIA Whittle™ (ex GEMCOM Whittle™) (Gemcom, 2011), are based on these methods. Picard (1976) showed that the ultimate pit problem is equivalent to the *maximum closure problem*. Based on this fact, Hochbaum and Chen (2000) proposed to use existing efficient algorithms to tackle the ultimate pit problem. Following Picard and Hochbaum & Chen ideas, Amankwah (2014) extended it to the multi-period case, showing that multi-period open-pit mining problem can be solved as a maximum flow problem.

Caccetta and Hill (2003) proposed a model containing constraints on the mining extraction sequence; mining, milling, and refining capacities; grades of mill feed and concentrates; stockpiles; logistics; and various operational requirements such as minimum pit-bottom width and maximum vertical depth (Newman et al., 2010) and use a customized version of the *branch-and-cut* algorithm to solve it up to a few hundred of thousands of blocks. Bley et al. (2010) use a similar model but considering fixed cutoff grade and incorporating additional cuts based on the capacity constraints that strengthen the formulation of the problem, in the sense that the value of the linear relaxation provides a tighter bound. Unfortunately, it is not clear how to scale the technique for large instances, as the number of cuts may explode very quickly. This strategy is also used by Fricke (2006) in order to find inequalities that improve various integer formulations of the same model. Gaupp (2008) also reduces the number of variables by deriving minimum and maximum extraction time-periods for each block, therefore eliminating some of the variables and reducing the original MIP model size.

Bienstock and Zuckerberg (2010) use Lagrangian relaxation on all constraints, except the precedence constraints (in this case the problem reduces to the ultimate pit problem). They consider all types of constraints, but focus on the resolution of the linear relaxation only, for which again they report very good improvements in resolution time with regards to the standard LP solvers. Chicoisne et al. (2012) focus on the case of one destination and one capacity constraint per period, developing a customized algorithm for the linear relaxation and a heuristic based on topological sorting to obtain integer feasible solutions from it, reporting good solutions for the problem in large instances. Cullembine et al. (2011) propose a heuristic procedure using Lagrangian relaxation on lower and upper capacity constraints and a sliding time window strategy in which extraction variables for late periods are also relaxed while variables corresponding to early periods are fixed incrementally; they add an additional constraint in which the bottom of the pit must contain at least two adjacent blocks. Lambert and Newman (2013) solve a similar problem that Cullembine et al. (except two adjacent blocks constraints), employing a tailored Lagrangian relaxation, which uses information obtained while generating the initial solution to select a dualization scheme for the resource constraints.

Another approach to tackle large-scale problems is based on aggregation procedures. Dagdelen et al. (1986, 1999) and Ramazan et al. (2005) work on a model with fixed cut-off grades, upper and lower bounds for blending, but only upper bounds for the capacity. They aggregate blocks into what they call *fundamental-trees* and present a relatively small case. Boland et al. (2009) propose a different model, in which they aggregate blocks into what they call *bins*. The extraction of individual blocks is controlled with continuous variables, but binary variables are used at the bin level to impose slope constraints. Jélvez et al. (2013) use heuristics based on incremental and aggregation approaches in order to solve the open pit block scheduling problem. Their model considers upper and lower capacity constraints, but the application considers only upper bounds. Approaches not based in linear programming include *genetic algorithms* and *tabu search*. Zhang (2006) uses genetic algorithm combined with a block aggregation technique based upon topological sort to reduce of number of variables in the model. According to Amankwah (2011), a difficulty in the use of genetic algorithm to solve the mine planning problem is that of dealing with the constraints. The practical problems with aggregation procedure are the consequences of aggregation or how to subsequently disaggregate. Another approach is developed by Tabesh and Askari-Nasab (2011), who present an algorithm that aggregates blocks into mining units and uses tabu search to calibrate the number of final units. The resulting problem is then solved using standard integer programming algorithms. The aggregation technique is interesting, because it is based on a similarity index that considers attributes like rock type, ore grades and the distance between the blocks.

This paper is organized as follows: Section 2 introduces the mathematical model in the framework of linear binary optimization problems; Section 3 describes some realistic cases of different sizes and structures, to which the numerical resolution procedure is applied in Section 4. Finally in Section 5 we establish the main conclusions and the most promising ideas for further development.

2. Mathematical model

The problem described in the previous section is modeled using a MIP model. The model to be introduced is in many ways similar to those considered "classical" but with one difference: In order to model exposed resource a new variable type and constraints need to be introduced.

Let B be the set of blocks, each block having a certain number of attributes, as density, tonnage, ore grade, etc. These attributes permit to determine the economic value of every block in B. The slope requirements for the set of blocks are described by a set of precedence arcs $A \subset B \times B$, in such a manner that the pair $(i, j) \in A$ means that block i must be extracted by time t if block j needs to be extracted at time t.

In this model a decision of whether the extracted material should be sent to a processing plant or to the waste dump (variable cutoff grade) is included. For each block it is assumed that the tonnage τ_i, the ore grade λ_i and the net discounted value, given by $b_i^t - p_i^t$ if block i is sent to processing plant, and $-m_i^t$ if block i is sent to waste dump, are known.

For every period t, maximum limits on the amount of material that is mined (M^t), and on the amount of ore that is milled (P^t), are imposed. Moreover, in each period a minimum exposed reserve F^t made available for the start of the next period must be guaranteed. In order to prevent the model from selecting low ore-grade blocks as exposed reserve, a cutoff grade Λ_{cg} is used to differentiate eligible blocks. For this reason a selective ore grade is defined as

$$\lambda_i(\Lambda_{cg}) = \begin{cases} \lambda_i & if\ \lambda_i \geq \Lambda_{cg} \\ 0 & otherwise \end{cases}$$

Table 1 summarizes the indexes, sets and parameters used in the MIP model.

Table 1: Main notation used in the MIP model

Symbol	Description
B	Set of blocks
A	Set of precedence arcs
T	Time horizon (number of periods)
b_i^t	Discounted profit resulting from the mining of block i at time-period t
p_i^t	Cost of mining and processing block i at time-period t
m_i^t	Cost of mining block i at time-period t
M^t	Maximum mining capacity for time-period t
P^t	Maximum processing capacity for time-period t
F^t	Minimum exposed reserve required at time-period t
τ_i	Tonnage of block i
λ_i	Ore grade of block i
Λ_{cg}	Cutoff grade

Three types of variables are used in the model, all of them are binary. The first type is the variable associated to the extraction for processing purposes for each block

$$x_i^t = \begin{cases} 1 & \text{if block } i \text{ is extracted and processed at time } t \\ 0 & \text{otherwise} \end{cases}$$

The second variable type describes the decision relating to the disposal of a block by sending it to the waste dump

$$w_i^t = \begin{cases} 1 & \text{if block } i \text{ is extracted and sent to waste dump at time } t \\ 0 & \text{otherwise} \end{cases}$$

The third variable type is used to identify exposed blocks; throughout the paper it will indistinctively be called "visibility" or "exposure" variable

$$y_i^t = \begin{cases} 1 & \text{if block } i \text{ is exposed at time } t \\ 0 & \text{otherwise} \end{cases}$$

The objective function for the model is the usual maximization of Net Present Value (NPV). The formulation of the mathematical model is as follows:

$$(OPBSEM) \quad max \sum_{i \in B} \sum_{t=1}^{T} [(b_i^t - p_i^t) x_i^t - m_i^t w_i^t] \tag{1}$$

$$\sum_{t=1}^{T} (x_i^t + w_i^t) \leq 1 \qquad \forall\, i \in B \tag{2}$$

$$\sum_{i \in B} \tau_i (x_i^t + w_i^t) \leq M^t \qquad \forall\, t \in \{1, \dots, T\} \tag{3}$$

$$\sum_{i \in B} \tau_i x_i^t \leq P^t \qquad \forall\, t \in \{1, \dots, T\} \tag{4}$$

$$y_j^t + \sum_{s=1}^{t} \left(x_j^s + w_j^s\right) \leq \sum_{s=1}^{t} (x_i^s + w_i^s) \qquad \forall\, (i,j) \in A,\;\; t \in \{1, \dots, T\} \tag{5}$$

$$y_i^t \leq x_i^{t+1} \qquad \forall\, i \in B,\;\; t \in \{1, \dots, T-1\} \tag{6}$$

$$\sum_{i \in B} \tau_i \lambda_i (\Lambda_{cg})\, y_i^t \geq \; F^t \qquad \forall\, t \in \{1, \dots, T-1\} \tag{7}$$

$$x_i^t, w_i^t, y_i^t \; \in \{0,1\} \qquad \forall\, i \in B,\;\; t \in \{1, \dots, T\} \tag{8}$$

The objective function (1) represents the maximization of the discounted cash flow. Constraint (2) simply expresses that it is not possible to choose two different destinations for a block, i.e., a block can be either sent to process or sent to the waste dump but not both at the same time. Constraint (3) and (4) establish an upper bound on mining and ore production for each period. Analogous constraints related to other capacities of the system could also be established (e.g. water, energy, mining capacity, etc.). Constraint (5) is the usual slope constraint of open pit planning models but written in a manner consistent with the identification of blocks that can be declared “exposed” for the start of the next period. Constraint (6) ensures that once exposed a block needs to be extracted and sent to processing plant at next time-period. Constraint (7) ensures that a minimum exposed reserve (in terms of units of extractable metal) must be available for the start of the next period. Finally, constraint (8) declares the nature of the variables involved in the model.

3. Numerical experiments

In this section a description of the application to some instances of the open-pit block scheduling problem with exposed mineral (OPBSEM) is provided. The numerical experiments have one objective in mind: To compare the performance of the proposed model with others strategies, such as (i) mine planning commercial software, and (ii) MIP models without exposed mineral, both in terms of extraction geometries, exposed mineral and (to a lesser extent) net present value (NPV). In the first subsection the implementation of the model on a hypothetical two-dimensional orebody is described. In the next subsection the dataset and parameters are presented and finally in the last subsection the experiments on more realistic three-dimensional case studies are described, their results with block scheduling obtained from other strategies will be compared, as mentioned earlier.

Example: Two-dimensional Dataset

The orebody considered herein is a two-dimensional orebody (a slice of three-dimensional deposit) that requires mining with a 45° slope angle. The block model contains 399 regular blocks and for each one it has attributes such as tonnage and cooper grade, also, economic values associated with the extraction and destination (processing plant or waste dump) of the block are given. The model decides the best destination for each block and defines exposed mineral for each period whilst maximizing the net present value of the entire project.

The planning horizon for this first case study is 3 years (considering annual time-periods). The discount rate is set to 10%. Mining and processing capacities are fixed to a maximum of 4M and 2.8M tons per year, respectively. For each period, a minimum of 12K tons of exposed reserve is required (as metal, calculated as tonnage multiplied by grade). The minimum grade for a block to be considered exposed ore reserve is set to 0.3%. The schedule obtained is shown in Figure 1.

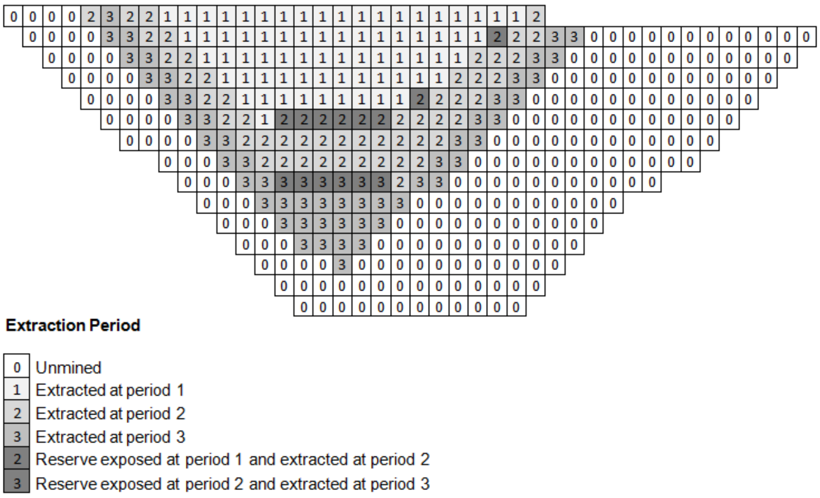

Figure 1: Block scheduling obtained through our exposed ore reserve model.

In the block scheduling, we can distinguish three important groups: Blocks not mined, which are represented by code 0 (white blocks); blocks mined in each period, encoded by numbers 1-2-3 (light gray), and exposed blocks within each period, in order to be mined and sent to processing at next period (dark grey blocks). An important aspect to highlight of schedule is the geometry obtained at the bottom of the pit per period where exposed mineral constrains applies, which is good in terms of operational spaces.

Dataset and parameters

The block models considered for this study were obtained from Minelib, a publicly available library of test problem instances for open-pit mining problems. The data comes from real-world mining projects and simulated data. Although open pit mining problems have appeared in literature dating back to the sixties, no standard representations exist, and there are no commonly available corresponding data sets. (Espinoza et al., 2013). Minelib is an effort to reduce this gap.

Newman1

The block model contains 1,060 regular blocks. For each block there are attributes such as rock type, tonnage, grade, and economic values associated the destination of the blocks (waste dump or processing plant). While there are details of how blocks were valued, unfortunately are not available all data to reproduce the block valuation. In this instance, the wall slope requirements are not given by an angle as usual, but to remove a given block, must first be extracted five blocks above, as is shown in the Figure 2.

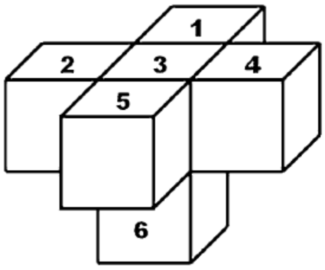

Figure 2: In order to remove block 6, must first be removed five blocks above (1, 2, 3, 4, 5). Source: Espinoza et al. (2013).

Annual time-periods with a yearly discount rate equivalent to a 8% are considered. The planning horizon is 6 (years), but 3 years will be enough to complete the exploitation. Mining and processing capacities are fixed to a maximum of 2M and 1.1M tons per year, respectively. For each period, a minimum of 7K tons of exposed reserve is required (as metal, calculated as tonnage multiplied by grade). The minimum grade for a block to be considered exposed ore reserve is set to 0.3%.

Marvin

This block model contains 53,271 blocks of $30 \times 30 \times 30$ meters, but can be reduced when blocks that are not accessible are removed from the mine. The reduced block model is inside a framework of 43 blocks in X, 51 blocks in Y and 15 blocks in Z, whose spatial coordinate ranges from 210m to 1290m in X, 300m to 1530m in Y and 30m to 450m in Z. The wall slope requirements are given by a 45° slope angle and using seven levels of precedence above a given block. This deposit has two metals of interest: copper and gold. For each block there are attributes such as tonnage, grades (copper and gold). In order to consider a single-element deposit instead of multi-element one, we use a copper equivalent grade:

$$EG_{cu} = \lambda^{cu} + \lambda^{au} \cdot \frac{P_{au} \cdot R_{au}}{P_{cu} \cdot R_{cu}}$$

where EG_{cu} is the copper equivalent grade, λ^{cu} and λ^{au} are the copper and gold grades, P_{cu} and P_{au} are the copper and gold prices, and R_{cu} and R_{au} are the copper and gold recoveries respectively. Table 2 contains some economical parameters used to get more realistic economic values for each block. The blocks' economic valuation was made according the following expression:

$$EV_i = [(P - C_s) \cdot f \cdot R \cdot \lambda_i - C_m^{ref} \cdot C_m^{caf} - C_p] \cdot \tau_i$$

where EV_i is the economic value of block i. P is the price of the element of interest. C_s, C_m^{ref} and C_p are the selling, reference mining and processing costs. C_m^{caf} is the block mining cost adjustment factor associated with the position (depth) of the block and f is an appropriated unit conversion factor. R is the recovery, λ_i is the equivalent ore grade and τ_i is the tonnage of block i. We consider annual time-periods with a yearly discount rate of 10%. The horizon planning is 7 years. Mining and processing capacities are fixed to a maximum of 60M and 20M tons per year, respectively. For each period, a minimum of 10K tons of exposed mineral is required (as metal, tonnage multiplied by cooper-grade).

The minimum grade for a block to be considered exposed ore reserve is set to 0.25%. Table 2 summarizes the main economic and technical parameters.

Table 2: Economic and technical parameters in Marvin

Parameter	Value	Unit
Copper price	3.02	$US\$/lb$
Gold price	1,132	$US\$/oz$
Copper recovery	0.88	$-$
Gold recovery	0.60	$-$
Selling cost	0.60	$US\$/lb$
Processing cost	10	$US\$/ton$
Reference mining cost	1.8	$US\$/ton$
Increment mining cost	0.002	$US\$/ton \cdot m$
Slope angle	45	$degrees$
Time horizon	7	$years$
Discount rate	0.10	$-$
Mining capacity	60,000,000	$Ton/year$
Processing capacity	20,000,000	$Ton/year$
Minimum exposed mineral (as metal)	100,000	$Ton/year$

Implementation/Instance

In this subsection further detail about the implementation of the exposed ore reserve model and others strategies to compare among them are provided. In the following cases no operational spatial constraints were considered. The following cases were implemented:

a) Open-pit block scheduling model with exposed ore reserve ($OPBSEM$) as detailed in section 2.

This is the main experiment in the article. The objective is to analyze the block scheduling obtained in terms of geometry extraction, exposed mineral and NPV. Newman and Marvin cases were implemented using the parameters explained in the previous subsection.

b) Open-pit block scheduling model without exposed mineral, that is, ($OPBSEM$) but without binary variable y_i^t and without constraints (6) and (7). This model is denoted as ($OPBS$).

This model considers precedence constraints and limited mining and processing capacities only. Newman and Marvin were implemented using the parameters (if corresponds) detailed in the subsection above. The comparison between this model and the exposed mineral model is interesting because it allows the evaluation of the insertion within the same formulation of the exposed ore reserve concept.

c) Bench-phase scheduling using mine-planning optimization software Whittle from Gemcom (currently Geovia).

Whittle is a traditional mine planning tool. Only the Marvin instance has been run, because the Newman database does not have some parameters required by Whittle, for example, recoveries. Thus, the parameters shown in Table 2 are used to set the instance (Marvin), except minimum exposed ore reserve requirement. Some important remarks: (i) Each block is associated with a single parcel only, (ii) the ore reserve selection method used is *cash flow*, (iii) no stockpiles are considered, (iv) the objective is to maximize the NPV. Bench-phase scheduling obtained can be transformed into a block scheduling: In the first scheduling, each bench-phase is composed of entire blocks and block fractions. However, if a fraction of a block is mined within a given period, then the remaining portion of the block will be mined in the next period. Therefore, there are blocks that are completely mined in two consecutive periods.

In order to implement a) and b) PuLP was used (Mitchell, 2009), which is free open source software written in Python. It is used to describe optimisation problems as mathematical models. PuLP was used in conjunction with GUROBI version 5.6.0 (Gurobi, 2013) to solve the resulting MIP models. Integer instances are solved up to a maximum 5% gap.

For implementing c), Whittle 4.5.2 was used.

In all cases the resolution of the instances was performed on an Intel Core i5-3570 CPU machine with 15.9 Gb running Windows XP version 2003. This machine has 4 processors with each running at 3.4 Ghz.

4. Results and discussion

In this section the results obtained from the numerical experiences introduced in the previous section are discussed. As mentioned therein, the interest is in studying differences between the geometries, schedules and production plans that arise from using the standard approach (Whittle and optimized block schedule) against the proposed model (optimized schedule with exposed ore reserve). While it is not the focus of the article, the economic impact of the difference techniques is also mentioned and a brief discussion about the computational effort required to solve the instances is included.

Newman case study

In the case of the Newman dataset, it was not possible to use Whittle because the block model lacks relevant information that forbids the application of the software, so it was only possible to compare the “classical” block scheduling optimization models with the proposed model variation.

Geometries

The pits obtained when scheduling the Newman case are shown in Figure 3. The figure presents projections on the X axis for both schedules,the colors correspond to the periods at which the blocks are extracted. There are three extraction periods (cyan, yellow and brown), unextracted blocks are in dark blue.

First notice that this dataset has a very special shape. It is not a box full with blocks, but it consists of 3 different disjoint parts. Also, the slopes at the borders are very steep. This is a property of the dataset considered, and has nothing to do with the schedules.

The most interesting property regarding the obtained geometries is that the ones with exposed ore reserve are quite better in terms of operational spaces and regularity. They are closer to a *worst case* and therefore suffer from fewer operational problems when designing phases later on.

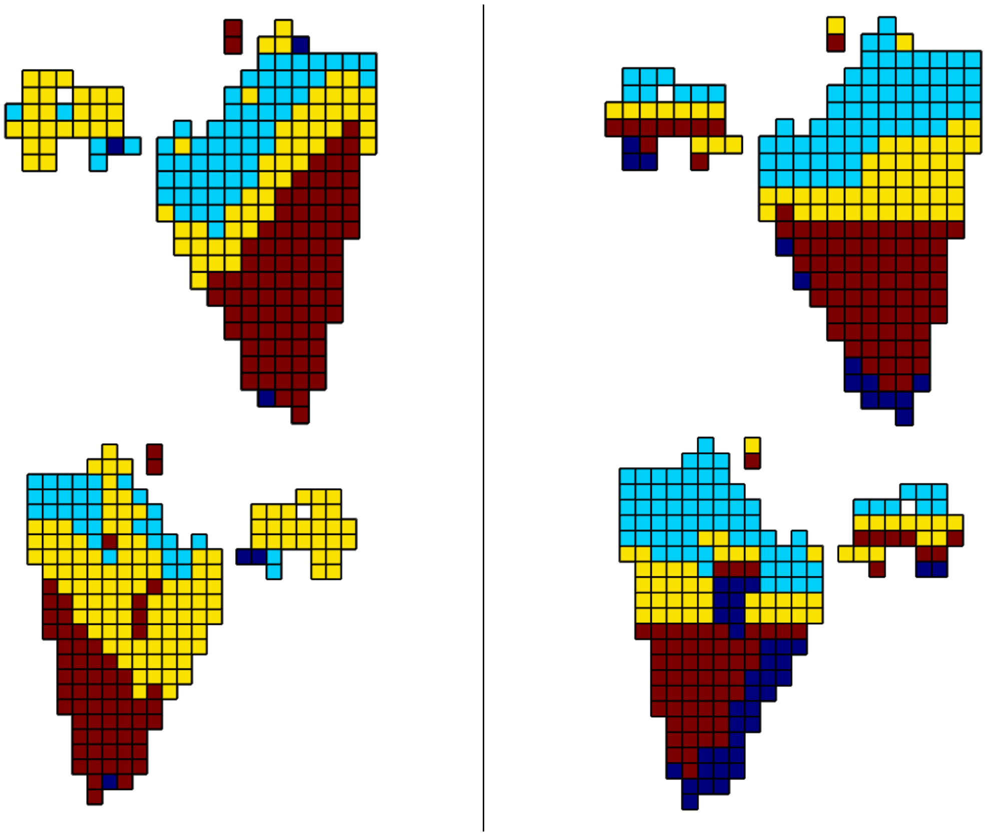

Figure 3: Pits by period of Newman case. On the left, geometries obtained using the standard OPBS formulation. On the right, geometries obtained using the exposed mineral formulation. Colors represent time periods.

A isometric view of the pits is also provided in Figure 4.

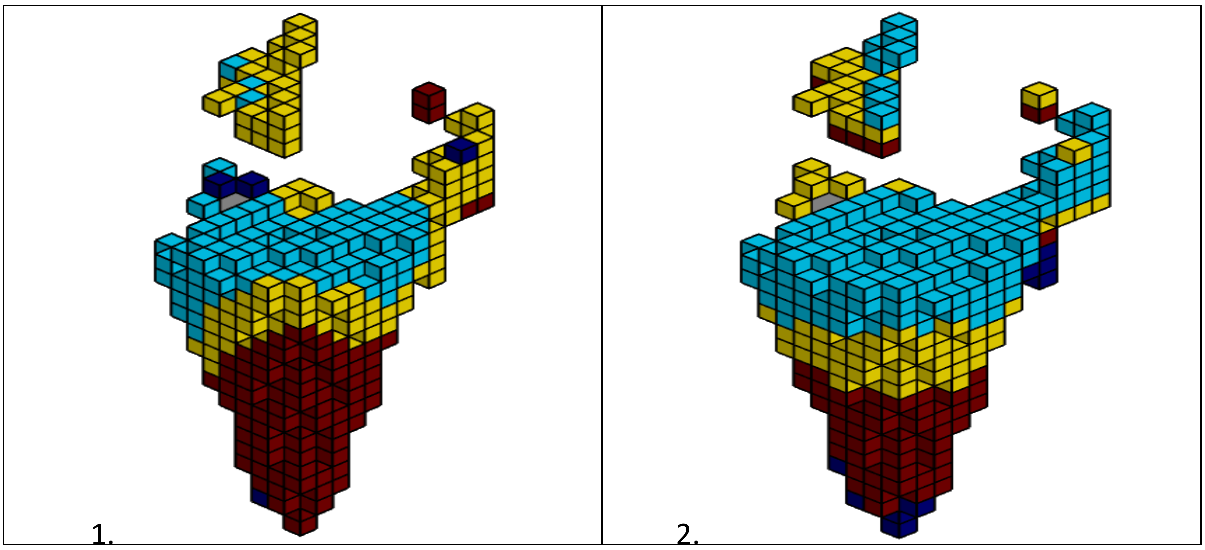

Figure 4: Isometric views of block schedules for the Newman dataset. On the left, the schedule using the standard OPBS formulation. On the right, the schedules obtained with the exposed mineral formulation. Colors represent time periods.

Production plans

The production plans for the schedules are presented in Table 3. As it is expected, the schedule proposed by OPBS extracts high value blocks as soon as possible, while OPBSEM tends to be more balanced. In terms of tonnage, the production plans are very similar and both saturate the mining capacity.

Table 3: Production Plans for the Newman dataset

	OPBS		OPBSEM	
Period	Grade	Tonnage	Grade	Tonnage
1	0.95	1,999,938	0.62	1,995,199
2	1.01	1,973,151	1.06	1,523,596
3	1.35	1,620,287	1.63	1,783,879
Totals		5,593,375		5,302,674

Marvin case study

The Marvin dataset provided the most complete example and therefore it was possible to use the block model to perform a comparison with results obtained from Whittle.

Geometries

Cuts for blocks X=750 are shown in Figure 5, for the three cases: Whittle, OPBS and OPBSEM.

First of all it is worth noting that for this case, Whittle reported that the best and worst case were the same, therefore, the schedule obtained by Whittle is simply a bench by bench extraction schedule as shown in the figure (on the left). The final pit obtained with Whittle is also a lot smaller than the ones obtained with the optimization models.

The pits obtained using the optimization models (middle and right side of Figure 5) are larger and less operational as they aim to maximize NPV and therefore to extract blocks with higher grades as soon as possible. Nevertheless, it is very interesting to observe that the schedule obtained using the model with exposed ore reserve does provide geometries that are more operational.

We also provide plant views of the schedules in Figure 6.

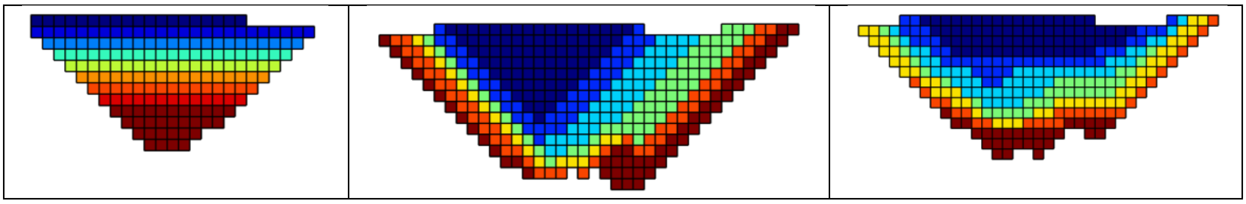

Figure 5: Block schedules for (from left to right) Whittle, OPBS and OPBSEM. Blocks are colored by extraction period from 1 (blue) to 7 (red). Cut corresponds to X=750.

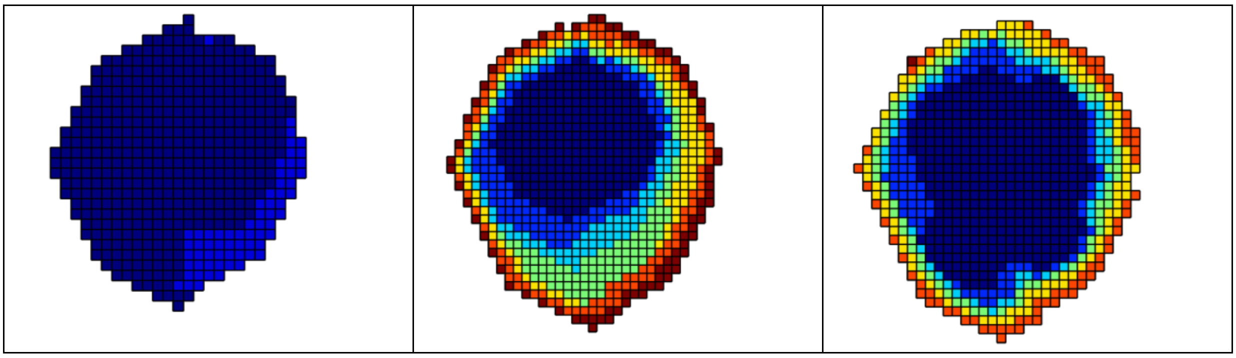

Figure 6: Plant view of schedules for the Marvin case study. Blocks are colored by extraction period from 1 (blue) to 7 (red).

Production plans

Production plans obtained with Whittle and models OPBS and OPBSEM are shown in Table 4.

Table 4: Comparison of Production Plans, Marvin Case Study

	Whittle		OPBS		OPBSEM	
Period	Grade	Tonnage	Grade	Tonnage	Grade	Tonnage
1	0.23	21,376,597	0.56	59,989,966	0.39	59,891,451
2	0.38	26,945,258	0.60	49,490,569	0.53	45,939,352
3	0.55	24,213,960	0.64	40,052,603	0.66	39,798,429
4	0.68	20,256,910	0.50	44,985,250	0.57	42,930,865
5	0.80	16,502,520	0.47	49,166,275	0.46	52,232,416
6	1.05	23,365,205	0.40	52,191,380	0.47	46,784,718
7	1.25	19,177,745	0.36	46,060,990	0.98	16,587,549
Totals		151,838,195		341,937,032		304,164,781

As reported in the previous subsection, the total tonnage of the solutions obtained with the optimization models is larger, actually more than double the size of the Whittle pit. It is very interesting also to see how the optimization models are able to take value from future periods to the initial ones, which can be seen in the behavior of the extracted grade. This is more than clear when the NPV of the three solutions are compared. For the Whittle solution it is US$ 2,003,363,988, for OPBS it gives US$ 3,047,498,282, and finally for OPBSEM the optimal NPV value is US$ 2,885,928,658.

Some comments on computational times

While the computational effort is not a focus on this paper, and indeed we expect it to become a research issue in future works, it is worth noting that the required time for solving these instances is still long for practical applications, as it is shown in Table 5.

Table 5: Time required to solve instances

	Whittle	OPBS	OPBSEM
Time [s]	-	28,605	126,040

5. Conclusions and open questions

In this paper, the concept of "exposed ore reserve" is introduced in a mathematical model and defined at a given period as the set of blocks for which all its precedent blocks have been already extracted but the block itself has been not. A new integer programming model to generate pit designs under exposed ore reserve requirements has been presented and tested in a set of standard instances.

First it was tested on a two-dimensional instance to validate the solutions provided by it with satisfactory results. Later three dimensional instances were attempted and comparisons between the model's output and existing models/tools were performed. The solutions obtained with the aid of the proposed model are consistent with those obtained by other optimization models for mine planning that do not have exposed ore reserve requirements in their formulation,

The new model having additional requirements than similar optimization models without the exposed ore reserve requirement, suffers from a reduction in the value of the solution; however, the geometrical nature of the new solutions obtained in the tests performed exhibit better operational spaces and regularity. And whilst it is true that it is not possible to generalize this property of the solutions to other instances, it is believed the model has the potential to produce results that are better suited to the type of operational spacing needed in mining operations, thus adding a valuable alternative to the current tools available to mine planners that could be a facilitator of better mine designs.

A potential research path to pursue in the future relates to the definition of the exposed ore reserve. At the present time the definition provided is a simple one that allows formulation into a model but it is believed that by no means it is exclusive. It is not known at the time of the writing if other definitions of exposed ore reserve are used in industry; nevertheless, exploration of alternative "exposure" definitions could prove fruitful in bringing designs of a more applicable nature. Other important questions for future study relate to assessing the suitability of the model for different types of deposits and mining operations, and which conditions are required in order for the model to produce solutions with the geometrical properties that have been observed so far in the present study.

The proposed model exhibits great potential in terms of applicability, but further algorithmic research is required to improve the computational execution time in the case of very large instances. However, it is important to mention here that the present paper has focused on introducing the concept and presenting the associated model. As the results obtained in the instances tested are encouraging enough to warrant additional research, it is believed that the first step taken in this study will serve as a solid foundation to new ways of thinking mine planning to the benefit on industry and practitioners.

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