

Fano resonances in waveguide arrays with saturable nonlinearity

Uta Naether,* Daniel E. Rivas, Manuel A. Larenas, Mario I. Molina, and Rodrigo A. Vicencio

Departamento de Física, Facultad de Ciencias, and Centro de Óptica y Fotónica (CEFOP), Universidad de Chile, Casilla 653, Santiago, Chile

*Corresponding author: unaether@u.chile.cl

Received May 21, 2009; revised July 27, 2009; accepted July 29, 2009; posted August 18, 2009 (Doc. ID 111749); published September 4, 2009

We study a waveguide array with an embedded nonlinear saturable impurity. We solve the impurity problem in closed form and find the nonlinear localized modes. Next, we consider the scattering of a small-amplitude plane wave by a nonlinear impurity mode, and discover regions in parameter space where transmission is fully suppressed. We relate these findings with Fano resonances and propose this setup as a means to control the transport of light across the array. © 2009 Optical Society of America
OCIS codes: 190.0190, 190.5530, 190.6135, 230.4320.

Discrete nonlinear systems have been studied in nonlinear optical waveguide arrays during the past 20 years [1,2]. One of its main advantages is the possibility to change and control all essential parameters, e.g., geometry, dimensionality, nonlinearity, beam angle, etc. On the other hand, scattering problems have always been of great interest to the physics community owing to their capacity to probe the physical nature of many complex systems. In the context of nonlinear waveguide arrays, the scattering of solitons against impurity potentials has shown a rich and complex phenomenology [3,4]. In addition, the scattering of plane waves by solitons has opened the possibility to observe Fano resonances [5]: nonlinearity generates several scattering channels, which can lead to resonances because of destructive interference and, as a consequence, to the total absence of transmission similar to the original Fano problem. As examples we can cite the prediction of Fano resonances in the context of nonlinear quadratic waveguide arrays [6] and also, in a very different research area, the recent prediction of similar resonances in Bose–Einstein condensates [7] (see [8] for a review of recent findings in this area).

In this Letter, we propose a saturable impurity embedded in a linear waveguide array [sketched in Fig. 1(a)] as what we believe to be a new possible experimental setup to observe Fano resonances. Recently, scattering of wide beams by bright and dark solitons has been experimentally studied in saturable nonlinear media [9] but, to the best of our knowledge, no direct observation of these types of resonances has been implemented yet. We also characterize the main properties of nonlinear impurity modes (NLMs) in this type of system and suggest the possibility of a switching-mode scheme, based on a judicious tuning of the system parameters.

We consider a saturable discrete nonlinear Schrödinger (s-DNLS) equation with a linear and a nonlinear defect at one site of the lattice ($n = n_c$):

$$-i \frac{du_n}{dz} = (u_{n+1} + u_{n-1}) + \left(\epsilon - \frac{\beta}{1 + |u_n|^2} \right) u_n \delta_{n,n_c}. \quad (1)$$

These coupled-mode equations describe the propagation of light in weakly coupled waveguides close to the first band of the bandgap structure. Amplitude u_n represents the light amplitude at a guide centered on site n , z is the propagation coordinate along the waveguides, ϵ is the strength of the linear defect, and β is the nonlinear coefficient. We include both types of defects, linear and nonlinear, to deal with a more general problem. A linear site impurity can be created by altering the geometry of a given waveguide, while at the same time tuning the spacing with its nearest neighbors in order to keep its coupling with the rest of the array unaltered. On the other hand, the nonlinear response at the impurity site can be boosted by a judicious amount of extra metal doping [10].

First, we look for stationary localized solutions centered at site n_c . We insert into Eq. (1) the ansatz $u_n(z) \equiv U_n(z) = U_0 x^{|n-n_c|} e^{i\lambda z}$ with U_0 , $x \in \mathbb{R}$. U_0 is the impurity-mode amplitude and x determines its localization length. By defining $g \equiv \beta/(1 + U_0^2)$ and imposing $|x| < 1$ we obtain $x = (g - \epsilon)/2 \pm \sqrt{1 + ((g - \epsilon)/2)^2}$ and $\lambda = \pm \sqrt{4 + (g - \epsilon)^2}$. The relation between g and ϵ determines the sign of λ . For $g > \epsilon$ ($g < \epsilon$) the sign is minus (plus), x and $\lambda < 0$ (x and $\lambda > 0$), and the solution is

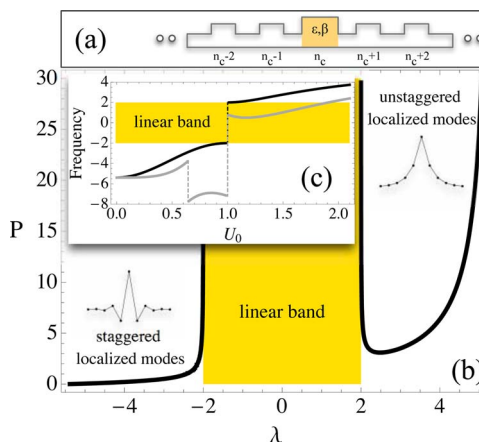


Fig. 1. (Color online) (a) Impurity setup. (b) P versus λ diagram for NLM. (c) λ versus U_0 for NLM (black curves) and ω_{cc} versus U_0 for LLM (gray curves).

staggered (unstaggered). The optical power is defined as $P \equiv \sum_n |u_n|^2 = U_0^2 \sqrt{4 + (g - \epsilon)^2} / |g - \epsilon|$. Hereafter, we fix $\beta = 10$ and $\epsilon = 5$ (thus, if $U_0 = 1 \rightarrow g = \epsilon$). Figure 1 shows power versus propagation constant of the impurity mode. From Fig. 1(b) we see that there is no power threshold for staggered modes (which are always linearly stable). On the other hand, unstaggered modes possess a threshold that separates two regimes: $\partial P / \partial \lambda < 0$ unstable and $\partial P / \partial \lambda > 0$ stable solutions. When nonlinear modes approach the linear band ($|g - \epsilon| \rightarrow 0$), the optical power diverges. This amplitude-dependent “transition” implies that, for a single saturable impurity, one can switch from a staggered to an unstaggered mode just by varying the input power. By using $u_n(0) = U_0 \delta_{n,n_c}$ as an initial condition for model (1), we have numerically checked that such a transition occurs for $U_0 \approx 2$. Below this amplitude, the mode possesses a π -phase shift between neighbor sites and above such values, there is no phase difference.

Now, we look for Fano resonances; i.e., what would happen if a small-amplitude plane wave is scattered by a NLM? In the impurity region, where the interaction takes place, a linear localized mode (LLM) can be generated. This mode corresponds to an extra channel, which is opened by the interaction. When this mode is fully excited, a zero transmission of plane waves can occur due to destructive interference and a Fano resonance appears [8]. We assume the plane wave amplitude $\phi_n(z)$ to be much smaller than the impurity-mode amplitude, i.e., $|\phi_0| \ll |U_0|$. We insert $u_n(z) = U_n(z) + \phi_n(z)$ and linearize Eq. (1) with respect to $\phi_n(z)$, obtaining

$$-i \frac{d\phi_n}{dz} = (\phi_{n+1} + \phi_{n-1}) + \left[\left(\epsilon - \frac{g^2}{\beta} \right) \phi_n + \frac{g^2}{\beta} U_0^2 e^{2i\lambda z} \phi_n^* \right] \delta_{n,n_c}. \quad (2)$$

To solve this problem, we use the ansatz $\phi_n(z) = a_n e^{i\omega z} + b_n^* e^{i(2\lambda - \omega)z}$ in Eq. (2), obtaining two coupled discrete equations:

$$\omega a_n = (a_{n+1} + a_{n-1}) + [(\epsilon - g^2/\beta)a_n + (g^2 U_0^2/\beta)b_n] \delta_{n,n_c}, \quad (3)$$

$$(2\lambda - \omega)b_n = (b_{n+1} + b_{n-1}) + [(\epsilon - g^2/\beta)b_n + (g^2 U_0^2/\beta)a_n] \delta_{n,n_c}. \quad (4)$$

a_n corresponds to the open channel, i.e., a traveling plane wave with a frequency given by $\omega_k = 2 \cos k$. k is the plane wave vector, which, in an experiment, is related to the input angle. Therefore the linear band covers the region $[-2, 2]$. b_n corresponds to the closed channel, whose frequency is determined by the interaction. The resonance occurs when the open channel amplitude at the impurity site is zero and the LLM is fully excited. Therefore, we decouple Eq. (4) [taking $a_0 = 0$] and look for localized solutions of the form $b_n = b_0 y^{|n-n_c|}$ with $|y| < 1$. We find that the frequency of

this mode is $\omega_{cc} = 2\lambda \pm \sqrt{4 + (\epsilon - g^2/\beta)^2}$, where the plus sign holds if $\epsilon < g^2/\beta$ (minus sign otherwise). We notice that a resonance is possible only when the frequency of the open channel matches the frequency of the closed one. It can be proved that the necessary condition for having ω_{cc} inside the linear band is $\beta > \epsilon > 0$. In any other case ω_{cc} will be out of the band and no resonances will be observed. This condition also implies that the linear impurity is absolutely necessary for having Fano resonances in the present model. Figure 1(c) shows the existence region of the LLM as a function of U_0 . The frequency ω_{cc} lies inside the linear band approximately for $U_0 \in [1, 1.9]$ and, therefore, only in such a region can a plane wave excite a LLM and be totally reflected.

The scattering problem is studied by considering an incoming plane wave and an LLM,

$$a_n = \begin{cases} B e^{ik(n-n_c)} + D e^{-ik(n-n_c)} & n < n_c \\ F e^{ik(n-n_c)} & n \geq n_c \end{cases},$$

$$b_n = b_0 y^{|n-n_c|}.$$

By inserting this ansatz in Eqs. (3) and (4) at sites $n = n_c$, $n_c \pm 1$ we find $F = B + D$ and $\omega_k = 2 \cos k$. By solving the algebraic problem we get the transmission coefficient $T \equiv |F/B|^2$ in closed form as $T = 4 \sin^2 k / [4 \sin^2 k + \Omega^2(k)]$ where

$$\Omega(k) = \epsilon - \frac{g^2}{\beta} + \frac{g^4 U_0^4 / \beta}{(g^2/\beta) - \epsilon \pm 2\sqrt{(\lambda - \cos k)^2 - 1}}, \quad (5)$$

where the plus (minus) sign holds for $\lambda - \cos k > (<) 1$. By fixing β and ϵ , the remaining free parameters are the beam angle k and the amplitude U_0 . Figure 2 shows the transmission coefficient in terms of k and U_0 . First, we clearly see that $T = 0$ (Fano resonance) appears only in the range $U_0 \sim \{1, 1.9\}$ where the frequency of the open channel can coincide with the frequency of the closed channel. Second, resonances are possible only when the NLM is unstaggered ($U_0 > 1$). This picture suggests a high degree of controllability of the transmission coefficient: by choosing different input powers and/or different

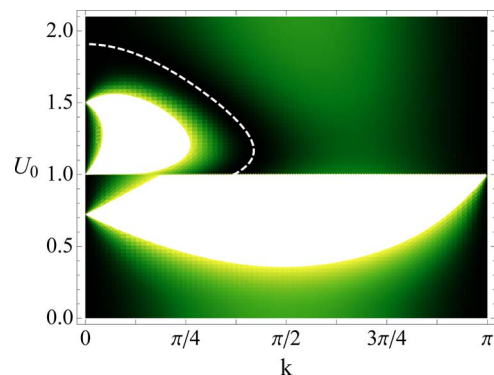


Fig. 2. (Color online) Transmission coefficient T as a function of beam angle k and amplitude U_0 . Bright (dark) regions denote high (low) T . The dashed curve marks places where $\omega_k = \omega_{cc}$.

input beam angles, we can efficiently control the amount of light going through or reflected back from the impurity region.

We perform numerical simulations of Eq. (1) to confirm our theoretical findings. As an initial condition, we take a localized NLM $U_0 x^{|n-n_i|}$ and an incoming plane wave initially centered at n_0 : $\phi_n = \phi_0 \exp[-\alpha(n-n_0)^2] \exp[ik(n-n_0)]$. The initial amplitude $\phi_0=0.01$ was chosen very small compared with U_0 to fulfill the analytical criteria. Also, $n_0 \ll n_c$ to avoid a possible initial overlap. We take $\alpha=0.001$ to provide a wide spatial distribution (~ 120 sites). By doing so, we can correctly simulate a “plane wave” with a well defined k . In Fig. 3 we show results for three different values of U_0 . The agreement between the theoretical T (curves) and direct numerical simulations (symbols) is excellent. This result validates our theory and provides good support for observing this phenomenon in real experimental setups. Below the critical amplitude $U_0=1$, no resonance exists because no local mode can be excited. The transmission profile is similar to the one found for the scattering of a plane wave against a nonlinear impurity ($U_0=0$) [11]. For $U_0>1$, the observation of Fano resonances is allowed. However, the stability of the nonlinear mode should also be considered. A small-amplitude plane wave can be viewed as a linear perturbation, therefore, the NLM has to be stable in order to numerically (and experimentally) observe the resonance. For $U_0=1.36$ the NLM is stable, and our theoretical prediction [red (diamonds) curve] matches perfectly the numerical computation (diamonds). Finally, for large amplitudes, no resonances are possible because ω_{cc} lies outside the band. The NLM grows in power and becomes an effective wall for the plane wave. Therefore, total absence of transmission is expected for large U_0 . Different β and ϵ values give a different phenomenology, i.e., a different transmission curve and resonance position. However, once we satisfy the condition $\beta > \epsilon > 0$ we can assure the existence of an LLM inside the linear band and, therefore, the possibility for observing a Fano resonance.

Finally, by fixing the amplitude $U_0=1.36$, we construct the output profile for different angles (see Fig.

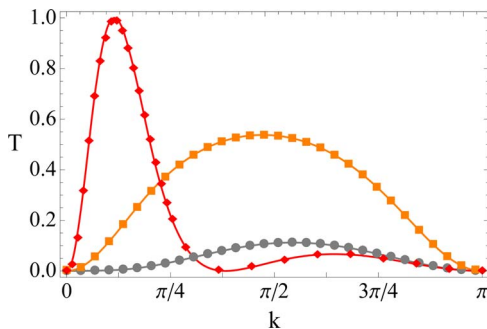


Fig. 3. (Color online) T versus k for $U_0=0.5$ (squares), 1.36 (diamonds), and 2 (circles). Curves correspond to theoretical T and symbols to numerical simulations.

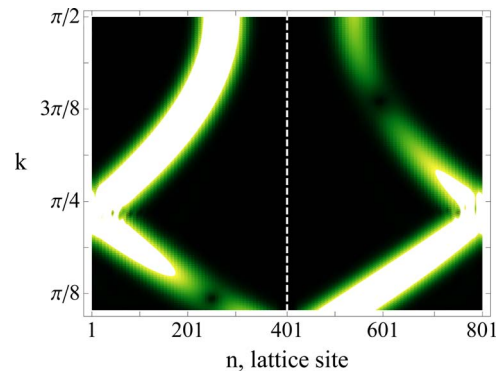


Fig. 4. (Color online) Linear output profile for different k . The dashed line marks the position of the impurity site. $U_0=1.36$.

4). For small k values, the transmission is very high, achieving its maximum ($T=1$) around $\pi/8$. As k is increased, the transmission and reflection coefficients becomes of the same order of magnitude. For $k > \pi/4$, most of the energy is reflected, becoming a maximum exactly at the Fano resonance ($k \sim 3\pi/8$), where no transmitted light is observed.

In conclusion, we have proposed a possible setup for observing Fano resonances in optical waveguide arrays. We found a very good agreement between theory and numerical simulations showing that, in principle, this phenomenon could be observed in current experiments. We also described a phase switch for NLM that can be controlled with the input power. We believe that our findings constitute a good example of the potential usefulness in using waveguide arrays for all-optical communication systems.

The authors acknowledge financial support from FONDECYT grants 1070897 and 1080374, PFB 024/2009, and from CONICYT fellowships.

References

1. F. Lederer, G. I. Stegeman, D. N. Christodoulides, G. Assanto, M. Segev, and Y. Silberberg, *Phys. Rep.* **463**, 1 (2008).
2. S. Flach and A. V. Gorbach, *Phys. Rep.* **467**, 1 (2008).
3. L. Morales-Molina and R. A. Vicencio, *Opt. Lett.* **31**, 966 (2006).
4. Y. Linzon, R. Morandotti, M. Volatier, V. Aimez, R. Ares, and S. Bar-Ad, *Phys. Rev. Lett.* **99**, 133901 (2007).
5. U. Fano, *Phys. Rev.* **124**, 1866 (1961).
6. A. E. Miroshnichenko, Y. S. Kivshar, R. A. Vicencio, and M. I. Molina, *Opt. Lett.* **30**, 872 (2005).
7. R. A. Vicencio, J. Brand, and S. Flach, *Phys. Rev. Lett.* **98**, 184102 (2007).
8. A. E. Miroshnichenko, S. Flach, and Y. S. Kivshar, arXiv:0902.3014.
9. E. Smirnov, C. E. Ruter, M. Stepic, V. Shandarov, and D. Kip, *Opt. Express* **14**, 11248 (2006).
10. M. Stepic, E. Smirnov, C. E. Ruter, L. Pronneke, D. Kip, and V. Shandarov, *Phys. Rev. E* **74**, 046614 (2006).
11. M. I. Molina, R. A. Vicencio, and Y. S. Kivshar, *Phys. Lett. A* **350**, 134 (2006).