Direct and inverse radiative transfer problems

How to go from stellar atmosphere modelling to aerosols problems?

Olivier Titaud
titaud@dim.uchile.cl

Postdoctorant
Centro de Modelmatiento Matemático
Universidad de Chile
Doctor in Numerical Analysis
Université of Saint Étienne, France
1. The transfer equation

2. Numerical resolution of the direct problem

3. Inverse problems
The transfer theory

**Studied physical process:** Propagation of a particles $p$ in a diluted discrete medium;

- $p[v] \rightarrow p[v']$ neutrons [speed].
- $p[v] \rightarrow p[v']$ photons [energy]
- $p[m] \rightarrow p[m']$ aerosols [mass]?

**The aim:** describing the radiative field by its properties at each position and time, depending on the direction and frequency.
The specific intensity

The main magnitude of the transfer theory is the Specific Intensity $I$.

- Measures the amount of energy carried throughout an elementary surface in a elementary direction cone and in an elementary frequency interval;

- Depends on: the position, the direction, the frequency and the time.
A simplified model

The medium is modelized by a finite slab: stratified with plane-parallel homogeneous layers static and in a steady state:

\[ z = 0 \]

\[ z = z^* \]

\[ \mu = \cos \theta \]

\[ I_0^{-}(\mu), \mu < 0 \]

\[ I_+^{+}(\mu), \mu > 0 \]

Examples: stellar atmospheres, clouds
The transfer equation

It describes the propagation of energy carried by a radiation:

For all \( z \in ]0, z^* [ , \mu \in ] - 1, 1 [ , \) and \( \nu > 0, \)

\[
\mu \frac{\partial I}{\partial z}(z, \mu, \nu) = -\chi(z, \nu)I(z, \mu, \nu) \\
+ \int_0^{+\infty} \sigma(z, \nu, \nu') J(z, \nu') d\nu' + E(z, \nu).
\]

\[
J(z, \nu') = \frac{1}{2} \int_{-1}^{1} I(z, \mu, \nu') d\mu : \text{mean intensity}
\]
The transfer equation

It describes the propagation of energy carried by a radiation:

For all $z \in ]0, z^* [$, $\mu \in ]-1, 1[$, and $\nu > 0$,

$$
\mu \frac{\partial I}{\partial z}(z, \mu, \nu) = -\chi(z, \nu)I(z, \mu, \nu)
+ \int_{0}^{+\infty} \sigma(z, \nu, \nu') J(z, \nu') \, d\nu' + E(z, \nu)
$$

Contribution of emission
The transfer equation

It describes the propagation of energy carried by a radiation:

For all \( z \in ]0, z^* [, \mu \in ]-1, 1[ , \) and \( \nu > 0 , \)

\[
\mu \frac{\partial I}{\partial z}(z, \mu, \nu) = -\chi(z, \nu)I(z, \mu, \nu)
+ \int_{0}^{+\infty} \sigma(z, \nu, \nu') J(z, \nu') d\nu' + E(z, \nu)
\]

Contribution of collisions
The transfer equation

It describes the propagation of energy carried by a radiation:

For all $z \in ]0, z^*[, \mu \in ]-1, 1[, and \nu > 0,$

$$
\mu \frac{\partial I}{\partial z} (z, \mu, \nu) = -\chi(z, \nu) I(z, \mu, \nu)
+ \int_{0}^{+\infty} \sigma(z, \nu, \nu') J(z, \nu') d\nu' + E(z, \nu).
$$

Opacity = absorption + scattering = extinction $> 0$
The transfer equation

It describes the propagation of energy carried by a radiation:

For all \( z \in ]0, z^* [ , \mu \in ]-1, 1[ , \text{ and } \nu > 0 , \)

\[
\mu \frac{\partial I(z, \mu, \nu)}{\partial z} = -\chi(z, \nu)I(z, \mu, \nu) + \int_{0}^{+\infty} \sigma(z, \nu, \nu') J(z, \nu') d\nu' + E(z, \nu). 
\]

Differential scattering coefficient.
Connection with transport equation

Transport equation:

\[
\frac{\partial f}{\partial t}(x, v, t) + v \cdot \nabla f(x, v, t) = Coll(f, x, v, t) + E(x, v, t)
\]

\(f\): velocity distribution function of propagation particles of energy \(\frac{1}{2}mv^2\).

Transfer equation: particles are photons of energy \(hv\).

\(v\) \quad \rightarrow \quad (\nu, s \equiv \frac{v}{|v|})

\(f(x, s, \nu, t)\) \quad \rightarrow \quad \left(\frac{1}{ch\nu}\right)I(x, s, \nu, t)

aerosols[mass] \quad \rightarrow \quad ?
Changing of variable

\[ \tau_\nu(z) = \int_z^{z^*} \chi_\nu(z') \, dz' : \text{optical depth} \]

\[ \tau_\nu(0) =: \tau_{\nu}^* : \text{optical thickness} \]

The optical thickness represents the difficulty to go through the medium:

- Transition L\(\alpha\) : \(\tau_{\nu}^* = 2.146 \times 10^{11}\)
- Transition H\(\alpha\) : \(\tau_{\nu}^* = 3.851 \times 10^5\)
- Continuum spectrum : \(\tau_{\nu}^* = 7.445\)
Changing of variable

\[ \mu \frac{\partial I}{\partial \tau}(\tau, \mu) = I(\tau, \mu) - \varpi(\tau) \int_{-1}^{1} I(\tau, \mu) \, d\mu - f(\tau) \]

\[ f(\tau) = \frac{E(\tau)}{\chi(\tau)} : \text{primary source function} \]

\[ \varpi(\tau) = \frac{\sigma(\tau)}{\chi(\tau)} : \text{albedo : probability of scattering photons} \]

Transition L\(\alpha\) : \(0.99 \leq \varpi(\tau) < 1\)

Transition H\(\alpha\) : \(2 \times 10^{-1} \leq \varpi(\tau) < 1\)

Continuum spectrum : \(2 \times 10^{-4} \leq \varpi(\tau) \leq 1\)
The specific intensity of the incident beam on the boundaries is known:

$$\tau = \tau^*$$

$$\tau = 0$$

$$I^+(\mu)$$

$$I^- (\mu)$$

$$I(0, \mu) = I^-(\mu) \quad \mu \in [-1, 0[$$

$$I(\tau^*, \mu) = I^+(\mu) \quad \mu \in ]0, 1]$$
1. The transfer equation

2. Numerical resolution

3. Inverse problems
Reduction of computation

Profile of matrix of the solved linear system

$x$: coefficient of modulus up than $10^{-12}$ (more than 50%)

Can us zeroing the small coefficients?
Iterative refinement

$m = 1000$
Iterative refinement

Computation of an approximation $y^{(0)}$ on a coarse grid ($n << m$)

by
one of the previous approximation methods

$n = 20$

$m = 1000$
Iterative refinement

Computation of an approximation $y^{(0)}$ on a coarse grid ($n << m$)

by one of the previous approximation methods

$y^{(1)} \rightarrow y^{(2)} \rightarrow y^{(k)} \rightarrow$ iterative refinement of $y^{(0)}$

$\begin{align*}
\text{Graph of } y^{(0)} & \quad \text{with } n = 20 \\
\text{Graph of } y^{(1)} & \quad \text{with } m = 1000 
\end{align*}$
Iterative refinement

Computation of an approximation $y^{(0)}$ on a coarse grid ($n << m$)

by

one of the previous approximation methods

Resolution of the same linear system of rank $n$

$\begin{align*}
  y^{(0)} & \rightarrow y^{(1)} \rightarrow y^{(2)} \rightarrow \cdots \rightarrow y^{(k)} \\
n = 20 & \quad m = 1000
\end{align*}$
1. The transfer equation

2. Numerical resolution

3. Inverse problems
Known and measured magnitudes

Momentum of $I$:

$$J_n(\tau) = \int_{-1}^{1} \mu^n I(\tau, \mu) d\mu$$

<table>
<thead>
<tr>
<th>Magnitude</th>
<th>&quot;solved star&quot; (Sun)</th>
<th>Other stars</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I(\tau_\star, \mu)$</td>
<td>known $\forall \mu$</td>
<td>known $\forall \mu$</td>
</tr>
<tr>
<td>$I(0, \mu)$</td>
<td>measured for $\mu \geq 0$</td>
<td>unknown</td>
</tr>
<tr>
<td>$J_1(0)$</td>
<td>measured</td>
<td>measured</td>
</tr>
<tr>
<td>$J_1(\tau_\star)$</td>
<td>known</td>
<td>known</td>
</tr>
</tbody>
</table>

$\tau = \tau_\star$ : interior the star - atmosphere boundary

$\tau = 0$ : surface of the atmosphere - vacuum boundary
Inverse problems

First inverse problem:
Get the function $\varphi$ and the real $\tau_*$ when
- $I(\tau_*, \mu)$ and $I(0, \mu)$ are given for all $\mu$;
- $f$ is given.

Second inverse problem:
Get the function $\varphi$ and the real $\tau_*$ when
- $J_1(0)$ is given;
- $I(\tau_*, \mu)$ is given for all $\mu \in \mathcal{D}$
- $f$ is given.
References


References


References
