On a generalized perturbed sweeping process with nonregular sets

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Summary

1. Introduction and motivation
2. Position of the problem
3. Basic assumptions
4. An existence result for the GPSP
5. Some consequences
6. Uniqueness
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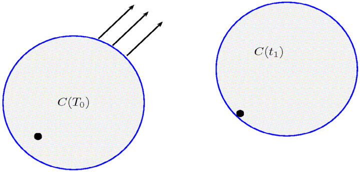
Introduction and motivation

Consider a large ring that contains a smaller ball inside, and the ring will start to move at time $t = T_0$.

Depending on the motion of the ring, the ball will just stay where it is (in case it is not hit by the ring), or otherwise it is swept towards the interior of the ring.

In this latter case the velocity of the ball has to point inwards to the ring in order not to leave.



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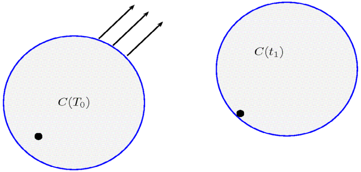

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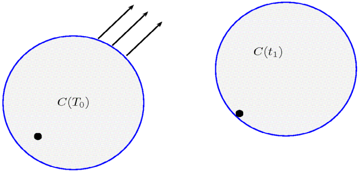

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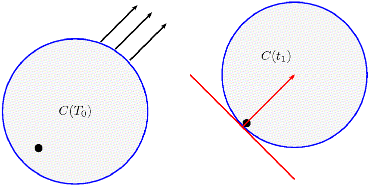

Introduction and motivation

Mathematically,

$$\begin{cases} -\dot{v}(t) \in N\left(C(t); v(t)\right) & \text{a.e. } t \in [T_0, T]; \\ v(T_0) = v_0 \in C(T_0), \end{cases} \tag{1.1}$$

where

- $v(t)$ is the position of the ball at time t.
- $C(t)$ is the moving set (the ring and its interior).
- $N\left(C(t); v(t)\right)$ is some appropriate outward normal cone of $C(t)$ at $v(t) \in C(t)$.

In the general setting, the set $C(t)$ is allowed to change its shape while moving.

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Introduction and motivation

Here we consider the Clarke normal cone: For $x \in S$

$$N(S;x) = \{\zeta \in H \colon \langle \zeta, v \rangle \leq 0 \, \forall v \in T_S(x)\},$$

where $T_S(x)$ is the Clarke tangent cone:

$$v \in T_S(x) \Leftrightarrow \forall x_i \to x, \forall t_i \to 0, \exists v_i \to v \text{ such that } x_i + t_i v_i \in S \, \forall i.$$

Also, we set $N(S,x) = \emptyset$ if $x \notin S$.

Moreau's perturbed sweeping process

$$\begin{cases} -\dot{v}(t) \in N\left(C(t); v(t)\right) + F(t, v(t)) & \text{a.e. } t \in [T_0, T]; \\ v(T_0) = v_0 \in C(T_0), \end{cases}$$

where

- $C\colon [T_0, T] \rightrightarrows H$ is a set-valued map with nonempty closed values.
- $N(S, \cdot)$ is the Clarke normal cone to S.
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It appears, for example, in:

- Granular materials (J.J. Moreau)
- Electrical circuits (V. Acary - B. Brogliato)
- Crowd motion (F. Bernicot - J. Venel)
- Hysteresis in elasto-plastic models (P. Krejči)

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Moreau's perturbed sweeping process: Existence theory

Main contributions:

- J.J. Moreau (1971 [28], 1972 [29], 1977 [30], 1999 [31])
  ▷ $C(t)$ convex and $F \equiv 0$.
- C. Castaing - T.D. Ha - M. Valadier (1993 [14])
  ▷ $C(t)$ convex and complement of a convex and F usc.
- M. Kunze - Monteiro-Marques (1996 [25], 2000 [27])
  ▷ $C(t)$ convex and $F \equiv 0$.
- H. Benabdellah (1999 [6])
  ▷$C(t)$ closed and $F \equiv 0$.
- G. Colombo - V. Goncharov (1999 [16])
  ▷ $C(t)$ closed and $F \equiv 0$.

Moreau's perturbed sweeping process: Existence theory

Main contributions:

- M. Bounkhel - L. Thibault (2005 [12])
  $\triangleright$ $C(t)$ prox-regular and F usc.
- J. Edmond - L. Thibault (2005 [17], 2006 [18])
  $\triangleright$ $C(t)$ prox-regular and F usc.
- T. Haddad - A. Jourani - L. Thibault (2008 [20])
  $\triangleright$ $C(t)$ α-far and F mixed usc.
- Thibault (2003 [33], 2008 [34], 2016 [35])
  $\triangleright$ $C(t)$ convex and prox-regular.
- A. Jourani - E. Vilches (2016 [24])
  $\triangleright$ $C(t)$ α-far and F usc.

State-dependent perturbed sweeping process

$$\begin{cases} -\dot{v}(t) \in N\left(C(t,v(t));v(t)\right) + F(t,v(t)) & \text{a.e. } t \in [T_0,T]; \\ v(T_0) = v_0 \in C(T_0,v_0), \end{cases}$$

where

1. $C\colon [T_0,T] \times H \rightrightarrows H$ is a set-valued map with nonempty closed values.
2. $N(S,\cdot)$ is the Clarke normal cone to S.
3. $F\colon [T_0,T] \times H \rightrightarrows H$ is a set-valued map with nonempty closed convex values satisfying some standard conditions.

State-dependent perturbed sweeping process: Existence theory

Main contributions:

- M. Kunze - M. Monteiro-Marques (1998 [26])
  ▷ $C(t,x)$ convex and $F \equiv 0$.
- N. Chemetov - M. Monteiro-Marques (2007 [15])
  ▷ $C(t,x)$ prox-regular and F continuous.
- M. Bounkhel - C. Castaing (2012 [11])
  ▷ $C(t,x)$ convex and $F \equiv 0$.
- T. Haddad (2013 [19])
  ▷ $C(t,x)$ convex and F usc.

State-dependent perturbed sweeping process: Existence theory

Main contributions:

- D. Azzam-Laouir - S. Izza - L. Thibault (2014 [5])
  ▷ $C(t,x)$ convex and F mixed usc.
- J. Noel - L. Thibault (2014 [32])
  ▷ $C(t,x)$ subsmooth and F usc.
- T. Haddad - I. Kecis - L. Thibault (2015 [21])
  ▷ $C(t,x)$ prox-regular and F mixed usc.
- A. Jourani - E. Vilches (2016 [22])
  ▷ $C(t,x)$ subsmooth and $F \equiv 0$.

Second-order perturbed sweeping process

$$\begin{cases} -\ddot{u}(t) \in N\left(C(t,u(t),\dot{u}(t));\dot{u}(t)\right) + F(t,u(t),\dot{u}(t)) & \text{a.e. } t \in [T_0,T]; \\ u(T_0) = u_0, \dot{u}(T_0) = v_0 \in C(T_0,u_0,v_0), \end{cases}$$

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Second-order perturbed sweeping process: Existence theory

Main contributions:

- C. Castaing (1988 [13])
  ▷ $C(t,u,v) = C(u)$ convex and $F \equiv 0$.
- M. Bounkhel et al (2003 [8] , 2004 [10], 2010 [9])
  ▷ $C(t,u,v) = C(u)$ prox-regular and $F(t,u,v) = F(t,v)$ usc.
- D. Azzam-Laouir et al (2008 [3], 2011 [4], 2014 [2])
  ▷ $C(t,u,v) = C(t)$ or $C(u)$ prox-regular and F usc.
- F. Bernicot - J. Venel (2012 [7])
  ▷ $C(t,u,v) = C(t)$ prox-regular and $F(t,u,v) = F(t,u)$ Lipschitz.
- S. Adly - B. Le (2016 [1])
  ▷ $C(t,u,v) = C(t,u)$ prox-regular and F usc.

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Generalized perturbed sweeping process (GPSP)

The *generalized perturbed sweeping process* (GPSP):

$$\begin{cases} -\dot{u}(t) = Bv(t) & \text{a.e. } t \in [T_0, T]; \\ -\dot{v}(t) \in N\left(C(t, u(t), v(t)); v(t)\right) + F(t, u(t), v(t)) + Au(t) & \text{a.e. } t \in [T_0, T]; \\ u(T_0) = u_0, v(T_0) = v_0 \in C(T_0, u_0, v_0), \end{cases}$$

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Why consider the GPSP?

- If $C(t,u,v) = C(t)$, $F(t,u,v) = F(t,v)$, $A = 0$ and $B = 0$ we recover the Moreau's perturbed sweeping process.
- If $C(t,u,v) = C(t,v)$, $F(t,u,v) = F(t,v)$, $A = 0$ and $B = 0$ we recover the state-dependent perturbed sweeping process.
- If $A = 0$ and $B = -I$ we recover the second-order perturbed sweeping process.

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Basic assumptions

$(\mathcal{H}^F)$: $F\colon [T_0, T] \times H \times H \rightrightarrows H$ has nonempty closed and convex values.

- For each $(u, v) \in H \times H$, $F(\cdot, u, v)$ is measurable.
- For a.e. $t \in [T_0, T]$, $F(t, \cdot, \cdot)$ is upper semicontinuous from $H \times H$ into H_w,
- There exist $c, d \in L^1(T_0, T)$ such that

$$d(0, F(t, u, v)) := \inf\{\|w\| : w \in F(t, u, v)\} \leq c(t)\|(u, v)\| + d(t),$$

for a.e. $t \in [T_0, T]$ and all $(u, v) \in H \times H$.

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for a.e. $t\in[T_0,T]$ and all $(u,v)\in H\times H$.

Basic assumptions

$(\mathcal{H}^F)$: $F\colon [T_0,T]\times H\times H \rightrightarrows H$ has nonempty closed and convex values.

- For each $(u,v)\in H\times H$, $F(\cdot,u,v)$ is measurable.
- For a.e. $t\in[T_0,T]$, $F(t,\cdot,\cdot)$ is upper semicontinuous from $H\times H$ into H_w,
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Basic assumptions (continued)

$(\mathcal{H}^C)$ $C\colon [T_0, T] \times H \times H \rightrightarrows H$ has nonempty closed values.

- There exist $\zeta \in \mathrm{AC}([T_0, T]; \mathbb{R})$, $L_1 \geq 0$ and $L_2 \in [0, 1[$ such that for all $s, t \in [T_0, T]$ and all $x, y, u, v \in H$

$$\mathrm{Hauss}(C(t,x,u), C(s,y,v)) \leq |\zeta(t) - \zeta(s)| + L_1\|x - y\| + L_2\|u - v\|.$$

- For every $t \in [T_0, T]$, every $r > 0$ and every pair of bounded sets $A, B \subset H$, the set $C(t, A, B) \cap r\mathbb{B}$ is relatively compact.

Basic assumptions (continued)

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Convex sets

Proposition

Let $S \subset H$ be a closed set. Then, S is convex if and only if

$$\langle x_1^* - x_2^*, x_1 - x_2 \rangle \geq 0,$$

holds for all $x_1, x_2 \in S$ and all $x_i^* \in N(S; x_i) \cap \mathbb{B}$ for $i = 1, 2$.

ρ-uniformly prox-regular sets

Definition

Let $S \subset H$ be a closed set. We say that S is *ρ-uniformly prox-regular* if

$$\langle x_1^* - x_2^*, x_1 - x_2 \rangle \geq -\frac{1}{\rho}\|x_1 - x_2\|^2,$$

holds for all $x_1, x_2 \in S$ and all $x_i^* \in N(S; x_i) \cap \mathbb{B}$ for $i = 1, 2$.

ρ-uniformly prox-regular sets

Proposition

Let $S \subset H$ be a closed set. Then S is ρ-uniformly prox-regular if and only if proj_S *is well defined and locally Lipschitz continuous on* $U_\rho(S) \cup S$.

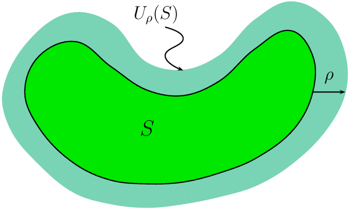

Figure: $U_\rho(S) := \{x \in H : 0 < d(x, S) < \rho\}$.

Uniformly subsmooth sets

Definition

S is *uniformly subsmooth*, if for every $\varepsilon > 0$ there exists $\delta > 0$, such that

$$\langle x_1^* - x_2^*, x_1 - x_2 \rangle \geq -\varepsilon \|x_1 - x_2\|,$$

holds for all $x_1, x_2 \in S$ satisfying $\|x_1 - x_2\| < \delta$ and all $x_i^* \in N(S; x_i) \cap \mathbb{B}$ for $i = 1, 2$.

equi-uniformly subsmooth sets

Definition

If $E \neq \emptyset$ the family $(S(t))_{t\in E}$ is *equi-uniformly subsmooth*, if for every $\varepsilon > 0$ there exists $\delta > 0$, such that for all $t \in E$

$$\langle x_1^* - x_2^*, x_1 - x_2 \rangle \geq -\varepsilon \|x_1 - x_2\|,$$

holds for all $x_1, x_2 \in S(t)$ satisfying $\|x_1 - x_2\| < \delta$ and all $x_i^* \in N\left(S(t); x_i\right) \cap \mathbb{B}$ for $i = 1, 2$.

Positively α-far sets

Definition

Let $\alpha \in]0,1]$. A set $S \subset H$ is *positively α-far* if there exists $\rho > 0$ such that if $x \in U_\rho(S)$ then the following implication holds:

$$\zeta \in \partial d_S(x) \quad \text{then} \quad \|\zeta\| \geq \alpha, \tag{3.1}$$

where $U_\rho(S) := \{x \in H \colon 0 < d(x,S) < \rho\}$ is the ρ-tube around S.

Moreover, if $E \neq \emptyset$, we say that the family $(S(t))_{t\in E}$ is *positively α-far* if every $S(t)$ satisfies (3.1) with the same α and the same $\rho > 0$.

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Moreover, if $E \neq \emptyset$, we say that the family $(S(t))_{t \in E}$ is *positively α-far* if every $S(t)$ satisfies (3.1) with the same α and the same $\rho > 0$.

Positively α-far sets

Proposition

Let $S \subset H$ be a ball-compact set.

1. *Assume that*

$$\langle x - \pi_1, x - \pi_2 \rangle \geq \alpha^2 d_S^2(x) \quad \forall x \in U_\rho(S),$$

for all $\pi_1, \pi_2 \in \mathrm{Proj}_S(x)$. *Then S is positively α-far.*

2. *If S is positively α-far then*

$$\langle x - \pi_1, x - \pi_2 \rangle \geq (2\alpha^2 - 1) d_S^2(x) \quad \forall x \in U_\rho(S),$$

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Relation between some classes

- If S is convex then S is positively 1-far (with $\rho = +\infty$).
- If S is ρ-uniformly prox-regular then S is positively 1-far (with the same ρ).
- If S is uniformly subsmooth then S is positively $\sqrt{1-\varepsilon}$-far for all $\varepsilon \in]0, 1[$.

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Positively α-far sets: An example

S is positively $\frac{\sqrt{2}}{2}$-far but not subsmooth.

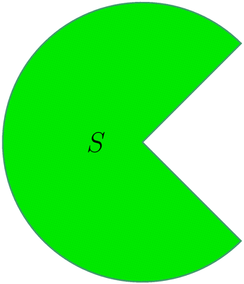

Figure: $S = \{(x,y) \in \mathbb{R}^2 \colon |y| \geq x\} \cap \mathbb{B}$

Subsmooth sets and sweeping process

Proposition

Assume that the following assumptions holds true:

- $\mathcal{H}^C$ *holds.*
- *The family* $\{C(t,u,v)\}_{\{(t,u,v)\in[T_0,T]\times H\times H\}}$ *is equi-uniformly subsmooth.*

Then, for all $t \in [T_0, T]$ *the set-valued map* $(u,v) \rightrightarrows \partial d(\cdot, C(t,u,v))(v)$ *is upper semicontinuous from* $H \times H$ *into* H_w.

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Reduction of sweeping process

To prove existence of the GPSP, we use the reduction technique, i.e.,

$$\begin{cases} -\dot{u}(t) = Bv(t) & \text{a.e. } t \in [T_0, T]; \\ -\dot{v}(t) \in N\left(C(t, u(t), v(t)); v(t)\right) & \\ \qquad\qquad + F(t, u(t), v(t)) + Au(t) & \text{a.e. } t \in [T_0, T]; \\ u(T_0) = u_0, v(T_0) = v_0 \in C(T_0, u_0, v_0). & \end{cases}$$

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$$\begin{cases} -\dot{u}(t) = Bv(t) & \text{a.e. } t \in [T_0, T]; \\ -\dot{v}(t) \in m(t, u(t), v(t))\partial d_{C(t,u(t),v(t))}(v(t)) & \\ \qquad\qquad + \tilde{F}(t, u(t), v(t)) + Au(t) & \text{a.e. } t \in [T_0, T]; \\ u(T_0) = u_0, v(T_0) = v_0 \in C(T_0, u_0, v_0), & \end{cases} \qquad (\mathcal{P}_{\text{Red}})$$

where $m(t, u, v)$ is a positive function and
$\tilde{F}(t, u, v) = F(t, u, v) \cap (c(t)\|(u, v)\| + d(t))\,\mathbb{B}$.

Reduction of sweeping process

By using the inclusion:

$$\partial d_S(x) \subseteq N(S;x) \cap \mathbb{B} \quad x \in S.$$

If we can prove that

$$v(t) \in C(t, u(t), v(t)) \text{ for all } t \in [T_0, T].$$

Then, any solution of $(\mathcal{P}_{\text{Red}})$ is a solution of GPSP.

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First main result

Theorem (Jourani-Vilches, 2016 [23])

Assume that the following assumptions hold true:

1. $(\mathcal{H}^F)$ *and* $(\mathcal{H}^C)$ *hold.*
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Then, there exists at least one solution of the GPSP:

$$\begin{cases} -\dot{u}(t) = Bv(t) & a.e.\ t\in[T_0,T];\\ -\dot{v}(t) \in N\left(C(t,u(t),v(t));v(t)\right) + F(t,u(t),v(t)) + Au(t) & a.e.\ t\in[T_0,T];\\ u(T_0)=u_0, v(T_0)=v_0\in C(T_0,u_0,v_0), \end{cases}$$

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Second main result

Theorem (Jourani-Vilches, 2016 [23])

Assume that the following assumptions hold true:

1. $(\mathcal{H}^F)$ *and* $(\mathcal{H}^C)$ *hold.*
2. *The family* $(C(t))_{\{t\in[T_0,T]\}}$ *is positively* α*-far.*

Then, there exists at least one solution of the GPSP:

$$\begin{cases} -\dot{u}(t) = Bv(t) & a.e.\ t \in [T_0,T]; \\ -\dot{v}(t) \in N\left(C(t); v(t)\right) + F(t,u(t),v(t)) + Au(t) & a.e.\ t \in [T_0,T]; \\ u(T_0) = u_0, v(T_0) = v_0 \in C(T_0), \end{cases}$$

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Moreau's perturbed sweeping process

Corollary

Assume that the following assumptions hold true:

- $(\mathcal{H}^F)$ *and* $(\mathcal{H}^C)$ *hold.*
- *The family* $(C(t))_{t\in[T_0,T]}$ *is uniformly positively* α*-far.*

Then, there exists at least one solution of

$$\begin{cases} -\dot{v}(t) \in N\left(C(t); v(t)\right) + F(t, v(t)) & a.e.\ t \in [T_0, T]; \\ v(T_0) = v_0 \in C(T_0). \end{cases}$$

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State-dependent sweeping process

Corollary

Assume that the following assumptions hold true:

- $(\mathcal{H}^F)$ *and* $(\mathcal{H}^C)$ *hold.*
- *The family* $\{C(t,v) \colon (t,v) \in [T_0,T] \times H\}$ *is equi-uniformly subsmooth.*

Then, there exists at least one solution of

$$\begin{cases} -\dot{v}(t) \in N\left(C(t,v(t));v(t)\right) + F(t,v(t)) & a.e.\ t \in [T_0,T]; \\ v(T_0) = v_0 \in C(T_0,v_0). \end{cases}$$

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Second-order sweeping process

Corollary

Assume that the following assumptions hold true:

- $(\mathcal{H}^F)$ *and* $(\mathcal{H}^C)$ *hold.*
- *The family* $\{C(t,u,v) \colon (t,u,v) \in [T_0,T] \times H \times H\}$ *is equi-uniformly subsmooth.*

Then, there exists at least one solution of

$$\begin{cases} -\ddot{u}(t) \in N\left(C(t,u(t),\dot{u}(t)); \dot{u}(t)\right) + F(t,u(t),\dot{u}(t)) & a.e.\ t \in [T_0,T]; \\ u(T_0) = u_0, \dot{u}(T_0) = v_0 \in C(T_0,u_0,v_0). \end{cases}$$

Second-order sweeping process

Corollary

Assume that the following assumptions hold true:

- $(\mathcal{H}^F)$ *and* $(\mathcal{H}^C)$ *hold.*
- *The family* $\{C(t,u,v)\colon (t,u,v) \in [T_0,T] \times H \times H\}$ *is equi-uniformly subsmooth.*

Then, there exists at least one solution of

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Assume that the following assumptions hold true:

- $(\mathcal{H}^F)$ *and* $(\mathcal{H}^C)$ *hold.*
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Uniqueness of Moreau's sweeping process

Let us consider the Moreau's sweeping process:

$$\begin{cases} -\dot{v}(t) \in N\left(C(t); v(t)\right) & \text{a.e. } t \in [T_0, T]; \\ v(T_0) = v_0 \in C(T_0), \end{cases}$$

It is known that if $C(t)$ is convex for all $t \in [T_0, T]$ then uniqueness hold.

Uniqueness of Moreau's sweeping process

Consider $v_1(t) = (-t/2, t/2)$ and $v_2(t) = (-t/2, -t/2)$ defined over $[0, 1]$. Then v_1 and v_2 are solutions of

$$\begin{cases} -\dot{v}(t) \in N\left(C(t); v(t)\right) & \text{a.e. } t \in [0,1]; \\ v(0) = (0,0) \in C(0), \end{cases}$$

where $C(t) = S - (t, 0)$ for $t \in [0, 1]$.

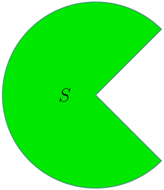

Figure: $S = \{(x, y) \in \mathbb{R}^2 : |y| \geq x\} \cap \mathbb{B}$

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On a generalized perturbed sweeping process with nonregular sets

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