On a generalized perturbed sweeping process with nonregular sets

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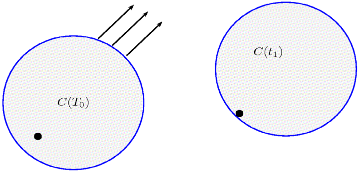
Summary

Introduction and motivation

Consider a large ring that contains a smaller ball inside, and the ring will start to move at time $t = T_0$.

Depending on the motion of the ring, the ball will just stay where it is (in case it is not hit by the ring), or otherwise it is swept towards the interior of the ring.

In this latter case the velocity of the ball has to point inwards to the ring in order not to leave.



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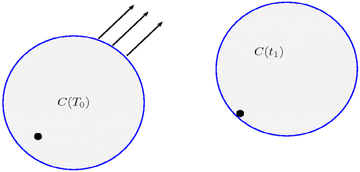

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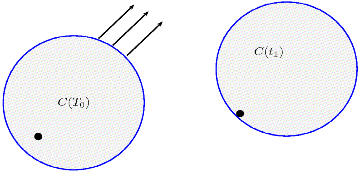

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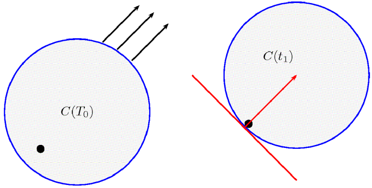

Introduction and motivation

Mathematically,

$$\begin{cases} -\dot{v}(t) \in N\left(C(t); v(t)\right) \ \text{ a.e. } t \in [T_0, T]; \\ v(T_0) = v_0 \in C(T_0), \end{cases} \tag{1.1}$$

where

- $v(t)$ is the position of the ball at time t.
- $C(t)$ is the moving set (the ring and its interior).
- $N\left(C(t); v(t)\right)$ is some appropriate outward normal cone of $C(t)$ at $v(t) \in C(t)$.

In the general setting, the set $C(t)$ is allowed to change its shape while moving.

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Introduction and motivation

Here we consider the Clarke normal cone: For $x \in S$

$$N\left(S;x\right) = \{\zeta \in H \colon \ \langle \zeta, v \rangle \leq 0 \, \forall v \in T_S(x)\},$$

where $T_S(x)$ is the Clarke tangent cone:

$$v \in T_S(x) \Leftrightarrow \forall x_i \to x, \forall t_i \to 0, \exists v_i \to v \text{ such that } x_i + t_i v_i \in S \, \forall i.$$

Also, we set $N(S,x) = \emptyset$ if $x \notin S$.

Moreau's perturbed sweeping process

$$\begin{cases} -\dot{v}(t) \in N\left(C(t); v(t)\right) + F(t, v(t)) & \text{a.e. } t \in [T_0, T]; \\ v(T_0) = v_0 \in C(T_0), \end{cases}$$

where

- $C\colon [T_0, T] \rightrightarrows H$ is a set-valued map with nonempty closed values.
- $N(S, \cdot)$ is the Clarke normal cone to S.
- $F\colon [T_0, T] \times H \rightrightarrows H$ is a set-valued map with nonempty closed convex values satisfying some standard conditions.

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It appears, for example, in:

- Granular materials (J.J. Moreau)
- Electrical circuits (V. Acary - B. Brogliato)
- Crowd motion (F. Bernicot - J. Venel)
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Moreau's perturbed sweeping process: Existence theory

Main contributions:

- J.J. Moreau (1971 [28], 1972 [29], 1977 [30], 1999 [31])
  $\triangleright$ $C(t)$ convex and $F \equiv 0$.
- C. Castaing - T.D. Ha - M. Valadier (1993 [14])
  $\triangleright$ $C(t)$ convex and complement of a convex and F usc.
- M. Kunze - Monteiro-Marques (1996 [25], 2000 [27])
  $\triangleright$ $C(t)$ convex and $F \equiv 0$.
- G. Colombo - V. Goncharov (1999 [16])
  $\triangleright$ $C(t)$ closed and $F \equiv 0$.
- H. Benabdellah (2000 [6])
  $\triangleright C(t)$ closed and $F \equiv 0$.

Moreau's perturbed sweeping process: Existence theory

Main contributions:

- M. Bounkhel - L. Thibault (2005 [12])
  ▷ $C(t)$ prox-regular and F usc.
- J. Edmond - L. Thibault (2005 [17], 2006 [18])
  ▷ $C(t)$ prox-regular and F usc.
- T. Haddad - A. Jourani - L. Thibault (2008 [20])
  ▷ $C(t)$ α-far and F mixed usc.
- Thibault (2003 [33], 2008 [34], 2016 [35])
  ▷ $C(t)$ convex and prox-regular.
- A. Jourani - E. Vilches (2016 [24])
  ▷ $C(t)$ α-far and F usc.

State-dependent perturbed sweeping process

$$\begin{cases} -\dot{v}(t) \in N\left(C(t, v(t)); v(t)\right) + F(t, v(t)) & \text{a.e. } t \in [T_0, T]; \\ v(T_0) = v_0 \in C(T_0, v_0), \end{cases}$$

where

1. $C\colon [T_0, T] \times H \rightrightarrows H$ is a set-valued map with nonempty closed values.
2. $N(S, \cdot)$ is the Clarke normal cone to S.
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State-dependent perturbed sweeping process: Existence theory

Main contributions:

- M. Kunze - M. Monteiro-Marques (1998 [26])
  ▷ $C(t,x)$ convex and $F \equiv 0$.
- N. Chemetov - M. Monteiro-Marques (2007 [15])
  ▷ $C(t,x)$ prox-regular and F continuous.
- M. Bounkhel - C. Castaing (2012 [11])
  ▷ $C(t,x)$ convex and $F \equiv 0$.
- T. Haddad (2013 [19])
  ▷ $C(t,x)$ convex and F usc.

State-dependent perturbed sweeping process: Existence theory

Main contributions:

- D. Azzam-Laouir - S. Izza - L. Thibault (2014 [5])
  ▷ $C(t,x)$ convex and F mixed usc.
- J. Noel - L. Thibault (2014 [32])
  ▷ $C(t,x)$ subsmooth and F usc.
- T. Haddad - I. Kecis - L. Thibault (2015 [21])
  ▷ $C(t,x)$ prox-regular and F mixed usc.
- A. Jourani - E. Vilches (2016 [22])
  ▷ $C(t,x)$ subsmooth and $F \equiv 0$.

Second-order perturbed sweeping process

$$\begin{cases} -\ddot{u}(t) \in N\left(C(t,u(t),\dot{u}(t));\dot{u}(t)\right) + F(t,u(t),\dot{u}(t)) & \text{a.e. } t \in [T_0,T]; \\ u(T_0) = u_0, \dot{u}(T_0) = v_0 \in C(T_0,u_0,v_0), \end{cases}$$

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Second-order perturbed sweeping process: Existence theory

Main contributions:

- C. Castaing (1988 [13])
  $\triangleright$ $C(t,u,v) = C(u)$ convex and $F \equiv 0$.
- M. Bounkhel et al (2003 [8] , 2004 [10], 2010 [9])
  $\triangleright$ $C(t,u,v) = C(u)$ prox-regular and $F(t,u,v) = F(t,v)$ usc.
- D. Azzam-Laouir et al (2008 [3], 2011 [4], 2014 [2])
  $\triangleright$ $C(t,u,v) = C(t)$ or $C(u)$ prox-regular and F usc.
- F. Bernicot - J. Venel (2012 [7])
  $\triangleright$ $C(t,u,v) = C(t)$ prox-regular and $F(t,u,v) = F(t,u)$ Lipschitz.
- S. Adly - B. Le (2016 [1])
  $\triangleright$ $C(t,u,v) = C(t,u)$ prox-regular and F usc.

Generalized perturbed sweeping process (GPSP)

The *generalized perturbed sweeping process* (GPSP):

$$\begin{cases} -\dot{u}(t) = Bv(t) & \text{a.e. } t \in [T_0, T]; \\ -\dot{v}(t) \in N\left(C(t, u(t), v(t)); v(t)\right) + F(t, u(t), v(t)) + Au(t) & \text{a.e. } t \in [T_0, T]; \\ u(T_0) = u_0, v(T_0) = v_0 \in C(T_0, u_0, v_0), \end{cases}$$

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- H is a separable Hilbert space.
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why consider the GPSP?

- If $C(t,u,v) = C(t)$, $F(t,u,v) = F(t,v)$, $A = 0$ and $B = 0$ we recover the Moreau's perturbed sweeping process.
- If $C(t,u,v) = C(t,v)$, $F(t,u,v) = F(t,v)$, $A = 0$ and $B = 0$ we recover the state-dependent perturbed sweeping process.
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Basic assumptions

$(\mathcal{H}^F)$: $F\colon [T_0, T] \times H \times H \rightrightarrows H$ has nonempty closed and convex values.

- For each $(u, v) \in H \times H$, $F(\cdot, u, v)$ is measurable.
- For a.e. $t \in [T_0, T]$, $F(t, \cdot, \cdot)$ is upper semicontinuous from $H \times H$ into H_w,
- There exist $c, d \in L^1(T_0, T)$ such that

$$d(0, F(t, u, v)) := \inf\{\|w\| : w \in F(t, u, v)\} \leq c(t)\|(u, v)\| + d(t),$$

for a.e. $t \in [T_0, T]$ and all $(u, v) \in H \times H$.

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$$d(0, F(t, u, v)) := \inf\{\|w\| : w \in F(t, u, v)\} \leq c(t)\|(u, v)\| + d(t),$$

for a.e. $t \in [T_0, T]$ and all $(u, v) \in H \times H$.

Basic assumptions

$(\mathcal{H}^F)$: $F\colon [T_0, T] \times H \times H \rightrightarrows H$ has nonempty closed and convex values.

- For each $(u, v) \in H \times H$, $F(\cdot, u, v)$ is measurable.
- For a.e. $t \in [T_0, T]$, $F(t, \cdot, \cdot)$ is upper semicontinuous from $H \times H$ into H_w,
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for a.e. $t \in [T_0, T]$ and all $(u, v) \in H \times H$.

Basic assumptions (continued)

$(\mathcal{H}^C)$ $C\colon [T_0, T] \times H \times H \rightrightarrows H$ has nonempty closed values.

- There exist $\zeta \in \mathrm{AC}([T_0, T]; \mathbb{R})$, $L_1 \geq 0$ and $L_2 \in [0, 1[$ such that for all $s, t \in [T_0, T]$ and all $x, y, u, v \in H$

$$\mathrm{Hauss}(C(t,x,u), C(s,y,v)) \leq |\zeta(t) - \zeta(s)| + L_1\|x - y\| + L_2\|u - v\|.$$

- For every $t \in [T_0, T]$, every $r > 0$ and every pair of bounded sets $A, B \subset H$, the set $C(t, A, B) \cap r\mathbb{B}$ is relatively compact.

Basic assumptions (continued)

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- For every $t \in [T_0, T]$, every $r > 0$ and every pair of bounded sets $A, B \subset H$, the set $C(t, A, B) \cap r\mathbb{B}$ is relatively compact.

Uniformly subsmooth sets

Definition

S is *uniformly subsmooth*, if for every $\varepsilon > 0$ there exists $\delta > 0$, such that

$$\langle x_1^* - x_2^*, x_1 - x_2 \rangle \geq -\varepsilon \|x_1 - x_2\|,$$

holds for all $x_1, x_2 \in S$ satisfying $\|x_1 - x_2\| < \delta$ and all $x_i^* \in N\left(S; x_i\right) \cap \mathbb{B}$ for $i = 1, 2$.

equi-uniformly subsmooth sets

Definition

If $E \neq \emptyset$ the family $(S(t))_{t\in E}$ is *equi-uniformly subsmooth*, if for every $\varepsilon > 0$ there exists $\delta > 0$, such that for all $t \in E$

$$\langle x_1^* - x_2^*, x_1 - x_2 \rangle \geq -\varepsilon \|x_1 - x_2\|,$$

holds for all $x_1, x_2 \in S(t)$ satisfying $\|x_1 - x_2\| < \delta$ and all $x_i^* \in N\left(S(t); x_i\right) \cap \mathbb{B}$ for $i = 1, 2$.

Positively α-far sets

Definition

Let $\alpha \in]0,1]$. A set $S \subset H$ is *positively α-far* if there exists $\rho > 0$ such that if $x \in U_\rho(S)$ then the following implication holds:

$$\zeta \in \partial d_S(x) \quad \text{then} \quad \|\zeta\| \geq \alpha, \tag{3.1}$$

where $U_\rho(S) := \{x \in H \colon 0 < d(x,S) < \rho\}$ is the ρ-tube around S.

Moreover, if $E \neq \emptyset$, we say that the family $(S(t))_{t\in E}$ is *positively α-far* if every $S(t)$ satisfies (3.1) with the same α and the same $\rho > 0$.

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Relation between some classes

- If S is convex then S is 1-far (with $\rho = +\infty$).
- If S is ρ-uniformly prox-regular then S is 1-far (with the same ρ).
- If S is uniformly subsmooth then S is $\sqrt{1-\varepsilon}$-far for all $\varepsilon \in]0,1[$.

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Positively α-far sets: An example

S is $\frac{\sqrt{2}}{2}$-far but not subsmooth.

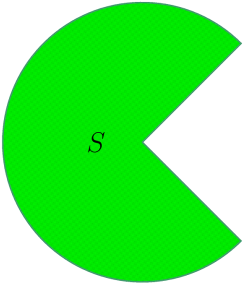

Figure: $S = \{(x, y) \in \mathbb{R}^2 \colon |y| \geq x\} \cap \mathbb{B}$

Subsmooth sets and sweeping process

Proposition

Assume that the following assumptions holds true:

- $\mathcal{H}^C$ *holds.*
- *The family* $\{C(t,u,v)\}_{\{(t,u,v)\in[T_0,T]\times H\times H\}}$ *is equi-uniformly subsmooth.*

Then, for all $t\in[T_0,T]$ *the set-valued map* $(u,v)\rightrightarrows\partial d(\cdot,C(t,u,v))(v)$ *is upper semicontinuous from* $H\times H$ *into* H_w.

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Reduction of sweeping process

To prove existence of the GPSP, we use the reduction technique, i.e.,

$$\begin{cases} -\dot{u}(t) = Bv(t) & \text{a.e. } t \in [T_0, T]; \\ -\dot{v}(t) \in N\left(C(t, u(t), v(t)); v(t)\right) & \\ \qquad\qquad + F(t, u(t), v(t)) + Au(t) & \text{a.e. } t \in [T_0, T]; \\ u(T_0) = u_0, v(T_0) = v_0 \in C(T_0, u_0, v_0). & \end{cases}$$

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$$\begin{cases} -\dot{u}(t) = Bv(t) & \text{a.e. } t \in [T_0, T]; \\ -\dot{v}(t) \in m(t, u(t), v(t))\partial d_{C(t,u(t),v(t))}(v(t)) & \\ \qquad\qquad + \tilde{F}(t, u(t), v(t)) + Au(t) & \text{a.e. } t \in [T_0, T]; \\ u(T_0) = u_0, v(T_0) = v_0 \in C(T_0, u_0, v_0), & \end{cases} \qquad (\mathcal{P}_{\text{Red}})$$

where $m(t, u, v)$ is a positive function and
$\tilde{F}(t, u, v) = F(t, u, v) \cap (c(t)\|(u, v)\| + d(t))\,\mathbb{B}$.

Reduction of sweeping process

By using the inclusion:

$$\partial d_S(x) \subseteq N(S;x) \cap \mathbb{B} \quad x \in S.$$

If we can prove that

$$v(t) \in C(t, u(t), v(t)) \text{ for all } t \in [T_0, T].$$

Then, any solution of $\mathcal{P}_{\text{Red}}$ is a solution of GPSP.

First main result

Theorem (Jourani-Vilches, 2016 [23])

Assume that the following assumptions hold true:

1. $(\mathcal{H}^F)$ *and* $(\mathcal{H}^C)$ *hold.*
2. *the family* $(C(t,u,v))_{\{(t,u,v)\in[T_0,T]\times H\times H\}}$ *is equi-uniformly subsmooth.*

Then, there exists at least one solution of the GPSP:

$$\begin{cases} -\dot{u}(t) = Bv(t) & a.e.\ t\in[T_0,T];\\ -\dot{v}(t) \in N\left(C(t,u(t),v(t));v(t)\right) + F(t,u(t),v(t)) + Au(t) & a.e.\ t\in[T_0,T];\\ u(T_0)=u_0, v(T_0)=v_0\in C(T_0,u_0,v_0), \end{cases}$$

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Second main result

Theorem (Jourani-Vilches, 2016 [23])

Assume that the following assumptions hold true:

1. $(\mathcal{H}^F)$ *and* $(\mathcal{H}^C)$ *hold.*
2. *The family* $(C(t))_{\{t\in[T_0,T]\}}$ *is positively* α*-far.*

Then, there exists at least one solution of the GPSP:

$$\begin{cases} -\dot{u}(t) = Bv(t) & a.e.\ t \in [T_0,T]; \\ -\dot{v}(t) \in N\left(C(t); v(t)\right) + F(t,u(t),v(t)) + Au(t) & a.e.\ t \in [T_0,T]; \\ u(T_0) = u_0, v(T_0) = v_0 \in C(T_0), \end{cases}$$

Moreau's perturbed sweeping process

Corollary

Assume that the following assumptions hold true:

- $(\mathcal{H}^F)$ *and* $(\mathcal{H}^C)$ *hold.*
- *The family* $(C(t))_{t\in[T_0,T]}$ *is uniformly positively* α*-far.*

Then, there exists at least one solution of

$$\begin{cases} -\dot{v}(t) \in N\left(C(t); v(t)\right) + F(t, v(t)) & a.e.\ t \in [T_0, T]; \\ v(T_0) = v_0 \in C(T_0). \end{cases}$$

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State-dependent sweeping process

Corollary

Assume that the following assumptions hold true:

- $(\mathcal{H}^F)$ *and* $(\mathcal{H}^C)$ *hold.*
- *The family* $\{C(t,v) : (t,v) \in [T_0,T] \times H\}$ *is equi-uniformly subsmooth.*

Then, there exists at least one solution of

$$\begin{cases} -\dot{v}(t) \in N\left(C(t,v(t)); v(t)\right) + F(t,v(t)) & a.e.\ t \in [T_0,T]; \\ v(T_0) = v_0 \in C(T_0,v_0). \end{cases}$$

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Second-order sweeping process

Corollary

Assume that the following assumptions hold true:

- $(\mathcal{H}^F)$ *and* $(\mathcal{H}^C)$ *hold.*
- *The family* $\{C(t,u,v)\colon (t,u,v)\in[T_0,T]\times H\times H\}$ *is equi-uniformly subsmooth.*

Then, there exists at least one solution of

$$\begin{cases} -\ddot{u}(t)\in N\left(C(t,u(t),\dot{u}(t));\dot{u}(t)\right)+F(t,u(t),\dot{u}(t)) & a.e.\ t\in[T_0,T];\\ u(T_0)=u_0, \dot{u}(T_0)=v_0\in C(T_0,u_0,v_0). \end{cases}$$

Second-order sweeping process

Corollary

Assume that the following assumptions hold true:

- $(\mathcal{H}^F)$ *and* $(\mathcal{H}^C)$ *hold.*
- *The family* $\{C(t,u,v) \colon (t,u,v) \in [T_0,T] \times H \times H\}$ *is equi-uniformly subsmooth.*

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$$\begin{cases} -\ddot{u}(t) \in N\left(C(t,u(t),\dot{u}(t)); \dot{u}(t)\right) + F(t,u(t),\dot{u}(t)) & a.e.\ t \in [T_0,T]; \\ u(T_0) = u_0, \dot{u}(T_0) = v_0 \in C(T_0,u_0,v_0). \end{cases}$$

Second-order sweeping process

Corollary

Assume that the following assumptions hold true:

- $(\mathcal{H}^F)$ *and* $(\mathcal{H}^C)$ *hold.*
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Uniqueness of Moreau's sweeping process

Let us consider the Moreau's sweeping process:

$$\begin{cases} -\dot{v}(t) \in N\left(C(t); v(t)\right) & \text{a.e. } t \in [T_0, T]; \\ v(T_0) = v_0 \in C(T_0), \end{cases}$$

It is known that if $C(t)$ is convex for all $t \in [T_0, T]$ then uniqueness hold.

Uniqueness of Moreau's sweeping process

Consider $v_1(t) = (-t/2, t/2)$ and $v_2(t) = (-t/2, -t/2)$ defined over $[0, 1]$. Then v_1 and v_2 are solutions of

$$\begin{cases} -\dot{v}(t) \in N\left(C(t); v(t)\right) & \text{a.e. } t \in [0,1]; \\ v(0) = (0,0) \in C(0), \end{cases}$$

where $C(t) = S - (t, 0)$ for $t \in [0, 1]$.

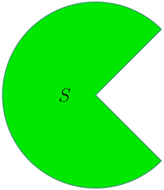

Figure: $S = \{(x, y) \in \mathbb{R}^2 \colon |y| \geq x\} \cap \mathbb{B}$

[1] S. Adly and B.K. Le.
Unbounded second-order state-dependent Moreau's sweeping processes in Hilbert spaces.
J. Optim. Theory Appl., 169(2):407–423, 2016.

[2] F. Aliouane and D. Azzam-Laouir.
A second order differential inclusion with proximal normal cone in Banach spaces.
Topol. Fixed Point Theory Appl., 44(1):143–160, 2014.

[3] D Azzam-Laouir.
Mixed semicontinuous perturbation of a second order nonconvex sweeping process.
Electron. J. Qual. Theory Differ. Equ., 37:1–9, 2008.

[4] D. Azzam-Laouir and S. Izza.
Existence of solutions for second-order perturbed nonconvex sweeping process.
Comput. Math. Appl., 62(4):1736–1744, 2011.

[5] D. Azzam-Laouir, S. Izza, and L. Thibault.
Mixed semicontinuous perturbation of nonconvex state-dependent sweeping process.
Set-Valued Var. Anal., 22(1):271–283, 2014.

[6] H. Benabdellah.
Existence of solutions to the nonconvex sweeping process.
J. Differential Equations, 164:286–295, 2000.

[7] F. Bernicot and J. Venel.
Existence of solutions for second-order differential inclusions involving proximal normal cones.
J. Math. Pures Appl., 98(3):257 – 294, 2012.

[8] M. Bounkhel.
General existence results for second order nonconvex sweeping process with unbounded perturbations.
Port. Math., 60(3):269–304, 2003.

[9] M. Bounkhel and R. Al-Yusof.
First and second order convex sweeping processes in reflexive smooth Banach spaces.
Set-Valued Var. Anal., 18(2):151–182, 2010.

[10] M. Bounkhel and D. Azzam-Laouir.
Existence results on the second-order nonconvex sweeping processes with perturbations.
Set-Valued Anal., 12(3):291–318, 2004.

[11] M. Bounkhel and C. Castaing.
State dependent sweeping process in p-uniformly smooth and q-uniformly convex Banach spaces.
Set-Valued Var. Anal., 20(2):187–201, 2012.

[12] M. Bounkhel and L. Thibault.
Nonconvex sweeping process and prox-regularity in Hilbert space.
J. Nonlinear Convex Anal., 6:359–374, 2005.

[13] C. Castaing.
Quelques problèmes d'évolution du second ordre. Exposé 5.
Sém. Anal. Convexe Montpellier, 1988.

[14] C. Castaing, T.X.D. Ha, and M. Valadier.
Evolution equations governed by sweeping process.
Set-Valued Anal., 1:109–139, 1993.

[15] N. Chemetov and M.D.P. Monteiro-Marques.
Nonconvex quasi-variational differential inclusions.
Set-Valued Anal., 15(3):209–221, 2007.

[16] G. Colombo and V.V. Goncharov.
The sweeping process without convexity.
Set-Valued Anal., 7:357–374, 1999.

[17] J.F. Edmond and L. Thibault.
Relaxation of an optimal control problem involving a perturbed sweeping process.
Math. Program., Ser. B, 104:347–373, 2005.

[18] J.F. Edmond and L. Thibault.
Bv solutions of nonconvex sweeping process differential inclusion with perturbation.
J. Differential Equations, 226(1):135 – 179, 2006.

[19] T. Haddad and T. Haddad.
State-dependent sweeping process with perturbation.
In George A. Anastassiou and Oktay Duman, editors, *Advances in Applied Mathematics and Approximation Theory*, volume 41 of *Springer Proceedings in Mathematics & Statistics*, pages 273–281. Springer New York, 2013.

[20] T. Haddad, A. Jourani, and L. Thibault.
Reduction of sweeping process to unconstrained differential inclusion.
Pac. J. Optim., 4:493–512, 2008.

[21] T. Haddad, I. Kecis, and L. Thibault.
Reduction of state dependent sweeping process to unconstrained differential inclusion.
J. Global Optim., 62(1):167–182, 2015.

[22] A. Jourani and E. Vilches.
Moreau-Yosida regularization of state-dependent sweeping processes with nonregular sets.
Submitted, 2016.

[23] A. Jourani and E. Vilches.
Partial galerkin method and generalized perturbed sweeping process with nonregular sets.
Submitted, 2016.

[24] A. Jourani and E. Vilches.
Positively α-far sets and existence results for generalized perturbed sweeping processes.
J. Convex Anal., 23(3), 2016.

[25] M. Kunze and M.D.P. Monteiro-Marques.
Yosida-Moreau regularization of sweeping processes with unbounded variation.
J. Differential Equations, 130(2):292–306, 1996.

[26] M. Kunze and M.D.P. Monteiro-Marques.
On parabolic quasi-variational inequalities and state-dependent sweeping processes.
Topol. Methods Nonlinear Anal., 12:179–191, 1998.

[27] M. Kunze and M.D.P Monteiro-Marques.
An introduction to Moreau's sweeping process.
In B. Brogliato, editor, *Impacts in Mechanical Systems*, volume 551 of *Lecture Notes in Physics*, pages 1–60. Springer Berlin Heidelberg, 2000.

[28] J.J. Moreau.
Rafle par un convexe variable I. Exposé 15.
Sém. Anal. Convexe Montpellier, 1971.

[29] J.J. Moreau.
Rafle par un convexe variable II. Exposé 3.
Sém. Anal. Convexe Montpellier, 1972.

[30] J.J. Moreau.
Evolution problem associated with a moving convex set in a Hilbert space.
J. Differential Equations, 26:347–374, 1977.

[31] J.J. Moreau.
Numerical aspects of the sweeping process.
Comput. Methods Appl. Mech. Engrg, 177(177):329–349, 1999.

[32] J. Noel and L. Thibault.
Nonconvex sweeping process with a moving set depending on the state.
Vietnam J. Math., 42(4):595–612, 2014.

[33] L. Thibault.
Sweeping process with regular and nonregular sets.
J. Differential Equations, 193(1):1–26, 2003.

[34] L. Thibault.
Regularization of nonconvex sweeping process in Hilbert space.
Set-Valued Anal., 16(2-3):319–333, 2008.

[35] L. Thibault.
Moreau sweeping process with bounded truncated retraction.
J. Convex Anal., 23(3), 2016.

On a generalized perturbed sweeping process with nonregular sets

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