Equations involving fractional Laplacian operator: Compactness and applications

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Abstract. In this paper, we consider the following problem involving fractional Laplacian operator:

$$(-\Delta)^\alpha u = |u|^{2^*_\alpha-2-\varepsilon}u + \lambda u \text{ in } \Omega, \quad u = 0 \text{ on } \partial\Omega, \tag{0.1}$$

where Ω is a smooth bounded domain in $\mathbb{R}^N$, $\varepsilon \in [0, 2^*_\alpha - 2)$, $0 < \alpha < 1$, $2^*_\alpha = \frac{2N}{N-2\alpha}$, and $(-\Delta)^\alpha$ is either the spectral fractional Laplacian or the restricted fractional Laplacian are considered in (0.1). We show for problem (0.1) with the spectral fractional Laplacian that for any sequence of solutions u_n of (0.1) corresponding to $\varepsilon_n \in [0, 2^*_\alpha - 2)$, satisfying $\|u_n\|_H \leq C$ in a certain Sobolev space H, u_n converges strongly in H provided that $N > 6\alpha$ and $\lambda > 0$. The same argument can also be used to obtain the same result for the restricted fractional Laplacian. An application of this compactness result is that problem (0.1) possesses infinitely many solutions under the same assumptions.