Boundary blow-up solutions of fractional equations in a measure framework

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In this talk, we discuss boundary blow-up solutions of fractional elliptic equations

$$
\begin{aligned}
(-\Delta)^\alpha u + g(u, |\nabla u|) = 0 \quad & \text{in} \quad \Omega, \\
u = 0 \quad & \text{in} \quad \mathbb{R}^N \setminus \bar{\Omega}, \\
\lim_{x\in\Omega, x\to\partial\Omega} u(x) = +\infty, &
\end{aligned}
\tag{0.1}
$$

where Ω is an open bounded C^2 domain in $\mathbb{R}^N$, the operator $(-\Delta)^\alpha$ denotes the fractional Laplacian with $\alpha \in (0,1)$. Let ω be the Hausdorff measure on $\partial\Omega$, we denote by $\frac{\partial^\alpha \omega}{\partial \vec{n}^\alpha}$ in the following sense

$$\langle \frac{\partial^\alpha \omega}{\partial \vec{n}^\alpha}, f\rangle = \int_{\partial\Omega} \frac{\partial^\alpha f(x)}{\partial \vec{n}_x^\alpha} d\omega(x), \quad \forall f \in C^\alpha(\bar{\Omega}),$$

where $\vec{n}_x$ is the unit inward normal vector of $\partial\Omega$ at point x.

Our idea is to study the fractional elliptic problem

$$
\begin{aligned}
(-\Delta)^\alpha u + g(u, |\nabla u|) = k\frac{\partial^\alpha \omega}{\partial \vec{n}^\alpha} \quad & \text{in} \quad \bar{\Omega}, \\
u = 0 \quad & \text{in} \quad \mathbb{R}^N \setminus \bar{\Omega}
\end{aligned}
\tag{0.2}
$$

with $k > 0$. We prove that problem (0.2) admits a nonnegative weak solution u_k when g is subcritical and u_k is a classical solution of problem (0.1).

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